Boris V.Vasiliev

Astrophysics

Boris V.Vasiliev

ASTROPHYSICS

and

astronomical measurement data

Annotation

Astrophysics = the star physics was beginning its development without a supporting of measurement data, which could not be obtained then. Still astrophysics exists without this support, although now astronomers collected a lot of valuable information. This is the main difference of astrophysics from all other branches of physics, for which foundations are measurement data. The creation of the theory of stars, which is based on the astronomical measurements data, is one of the main goals of modern astrophysics. Below the principal elements of star physics based on data of astronomical measurements are described. The theoretical description of a hot star interior is obtained. It explains the distribution of stars over their masses, mass-radius-temperature and mass-luminosity dependencies. The theory of the apsidal rotation of binary stars and the spectrum of solar oscillation is considered. All theoretical predictions are in a good agreement with the known measurement data, which confirms the validity of this consideration.

Contents

Chapter 1

Introduction

1.1 Astrophysics and astronomical measurements

"A question that sometimes
drives me hazy:
am I or are the others crazy?"
A.Einstein

It is obvious that the primary goal of modern astrophysics must be a creation of a star theory that can explain the existence of dependencies of parameters of stars and of the Sun, which are measured by astronomers.

The technical progress of astronomical measurements in the last decade has revealed the existence of different relationships that associate together the physical parameters of the stars.
To date, there are about a dozen of such new discovered relationships: it is dependencies of temperature-radius-luminosity-mass of stars, the spectra of seismic oscillations of the Sun, distribution of stars on mass, the dependence of the magnetic fields of stars from their moments and speeds of rotation, etc.

All these relationships are defined by phenomena which occurring inside stars. So the theory of the internal structure of stars should be based on these quantitative data as on boundary conditions.

Existing theories of stellar interiors can not explain of the new data. The modern astrophysics[1] prefers speculative considerations. It elaborates qualitative

[1]The modern astrophysics has a whole series of different branches. It is important to stress

theories of stars that are not pursued to such quantitative estimates, which could be compared with the data of astronomers. Everything is done in such a way as if the new astronomical data are absent. Of course, the astrophysical community knows about the existence of dependencies of stellar parameters which were measured by astronomers. However, in modern astrophysics it is accepted to think, that if an explanation of a dependency is not found, that it can be referred to the category of empirical one and it need no an explanation. The so-called empirical relations of stellar luminosities and temperatures - the Hertzsprung-Russell diagram - is known about the hundred years but its quantitative explanation is not found.

The reason that prevents to explain these relationships is due to the wrong choice of the basic postulates of modern astrophysics. Despite of the fact that all modern astrophysics believe that the stars consist from a plasma, it historically turned out that the theory of stellar interiors does not take into account the electric polarization of the plasma, which must occur within stars under the influence of their gravitational field. Modern astrophysics believes that the gravity-induced electric polarization (GIEP) of stellar plasma is small and it should not be taken into account in the calculations, as this polarization was not taken into account in the calculations at an early stage of development of astrophysics, when about a plasma structure of stars was not known. However, plasma is an electrically polarized substance, and an exclusion of the GIEP effect from the calculation is unwarranted. Moreover without of the taking into account of the GIEP-effect, the equilibrium stellar matter can not be correctly founded and a theory would not be able to explain the astronomical measurements. Accounting GIEP gives the theoretical explanation for the all observed dependence.

As shown below, the account of the gravity-induced electric polarization of the intra-stellar plasma gives possibility to develop a model of the star, in which all main parameters - the mass of the star, its temperature, radius and luminosity - are expressed by certain combinations of world constants and the individuality of stars is determined by only two parameters - the mass and charge number of nuclei, from which the plasma of these stars is composed. It gives the quantitatively and fairly accurate explanation of all dependencies, which were measured by astronomers.

The important feature of this stellar theory, which is built with the GIEP acconting, is the lack of a collapse in the final stage of the star development, as well as "black holes" that could be results from a such collapse. The main features of this concept were previously published in [1]-[3].

---

that all of them except the physics of hot stars beyond the scope of this consideration; we shall use the term "astrophysics" here and below in its initial meaning - as the physics of stars.

1.2 The basic postulate of astrophysics

We can assume that modern astrophysics emerged in the early twentieth century and milestone of this period was the work R. Emden ≪Die Gaskugeln≫. It has laid the basis for the description of stars as gas spheres. Gases can be characterized by different dependencies of their density from pressure, ie they can be described by different polytropes. According to Emden the equations of state of the gases producing the stars determine their characteristics - it can be either a dwarf, or a giant, or main sequence star, etc. The such approach to the description of stars determined the choice of postulates needed for the theory.

Any theory based on its system of postulates.

The first basic postulate of astrophysics - the Euler equation - was formulated in a mathematical form by L.Euler in a middle of 18th century for the "terrestrial" effects description. This equation determines the equilibrium condition of liquids or gases in a gravitational field:

$$\gamma \mathbf{g} = -\nabla P. \tag{1.1}$$

According to it the action of a gravity forth $\gamma \mathbf{g}$ (γ is density of substance, $\mathbf{g}$ is the gravity acceleration) in equilibrium is balanced by a forth which is induced by the pressure gradient in the substance.

All modern models of stellar interior are obtained on the base of the Euler equation. These models assume that pressure inside a star monotone increases depthward from the star surface. As a star interior substance can be considered as an ideal gas which pressure is proportional to its temperature and density, all astrophysical models predict more or less monotonous increasing of temperature and density of the star substance in the direction of the center of a star.

While we are talking about materials with atomic structure, there are no doubt about the validity of this equation and its applicability. This postulate is surely established and experimentally checked be "terrestrial" physics. It is the base of an operating of series of technical devices - balloons, bathyscaphes and other.

Another prominent astrophysicist first half of the twentieth century was A. Eddington. At this time I. Langmuir discovered the new state of matter - plasma. A.Eddington was first who realized the significance of this discovery for astrophysics. He showed that the stellar matter at the typical pressures and temperatures, should be in the plasma state.

1.3 The another postulate

The polarizability of atomic matter is negligible. [2]

There was not needs to take into account an electric polarization at a consideration of cosmic bodies which are composed by atomic gases.

[2] If you do not take account of ferroelectrics, piezoelectrics and other similar substances. Their consideration is not acceptable here.

But plasma is an electrically polarized substance.

It is necessary to take into account GIEP of intra-stellar plasma.

Therefore, at consideration of an equilibrium in the plasma, the term describing its possible electrical polarization $\mathfrak{P}$ should be saved in the Euler equation:

$$\gamma \mathbf{g} + \mathfrak{P}\nabla\mathfrak{P} + \nabla P = 0, \tag{1.2}$$

This leads to the possibility of the existence of a fundamentally new equilibrium state of stellar matter, at which it has a constant density and temperature:

$$\nabla P = 0$$

$$\gamma \mathbf{g} + \mathfrak{P}\nabla\mathfrak{P} = 0, \tag{1.3}$$

that radically distinguishes this equilibrium state from equilibrium, which is described by the Eq.(1.1).

1.3.1 Thus two postulates can be formulated. Which of these postulates is correct?

The general rule speaks for taking into account the effect of the polarization: at the beginning of determination of the equilibrium equations, one must consider all forces which, it seems, can influence it and only in the result of calculations discard small influences. However, this argument is not strong.

The method of false postulate rejecting was developed in the late Middle Ages, when this problem was sharply.[3]

The scientific approach to choosing the right postulates was developed by Galileo.

1.3.2 The Galileo's method

The modern physics begins its formation at last 16 c. - middle 17 c. mainly with works of W.Gilbert and G.Galileo. They introduce into practice the main instrument of the present exact science - the empirical testing of a scientific hypothesis. Until that time false scientific statements weren't afraid of an empirical testing. A flight of fancy was dainty and refined than an ordinary and crude material world. The exact correspondence to a check experiment was not necessary for a philosophical theory, it almost discredited the theory in the experts opinion. The discrepancy of a theory and observations was not confusing at that time.

[3] W. Gilbert, in his book ≪De magnete, magneticisque corparibus etc≫(1600) pointed out that only experiment can prove the fallacy of a number of judgments which are generally accepted in educated society . Without experimental verification, the common judgments can be often very strange.

Now the empirical testing of all theoretical hypotheses gets by a generally accepted obligatory method of the exact science. As a result all basic statements of physics are sure established and based on the solid foundation of an agreement with measurement data.

To solve the problem of the correct choice of the postulate, one has the Galileo's method. It consists of 3 steps:

(1) to postulate a hypothesis about the nature of the phenomenon, which is free from logical contradictions;

(2) on the base of this postulate, using the standard mathematical procedures, to conclude laws of the phenomenon;

(3) by means of empirical method to ensure, that the nature obeys these laws (or not) in reality, and to confirm (or not) the basic hypothesis.

The use of this method gives a possibility to reject false postulates and theories, provided there exist a necessary observation data, of course.

Let's see what makes this method in our case.
Both postulates are logically consistent - and (1.1), and (1.2).
The theory constructed on the basis of the first postulate is all modern astrophysics. There are a lot of laws that are good mutually agreed upon.

1.3.3 What does the the astronomic measurement data express?

Are there actually astronomic measurement data, which can give possibility to distinguish "correct" and "incorrect" postulates of stellar interior physics? What must one do, if the direct measurement of the star interior construction is impossible?

Previously such data were absent. They appeared only in the last decade. The technical progress of astronomical measurements in the last decade discovered that the physical parameters of the stars are related together.
However, these new data do not fit to models of modern astrophysics.

It seems clear to me that the primary goal of modern astrophysics is to create a theory that explains the dependencies of parameters of stars and of the Sun, which are measured by astronomers in recent decades.

1.4 About a star theory development

The following chapters will be devoted to the construction of the theory of stars with taking into account of the GIEP-effect (1.3) and comparisons of the resulting model with measurement data.

It will be shown below that all these dependencies obtain a quantitative explanation. At that all basic measuring parameters of stars - masses, radii, temperatures -

can be described by definite rations of world constants, and it gives a good agreement with measurement data.

The correct choice of the substance equilibrium equation is absolute requirement of an development of the star theory which can be in agreement with measuring data.

To simplify a task of formulation of the such theory , we can accept two additional postulates.

A hot star generates an energy into its central region continuously. At the same time this energy radiates from the star surface. This radiation is not in equilibrium relatively stellar substance. It is convenient to consider that the star is existing in its stationary state. It means that the star radiation has not any changing in the time, and at that the star must radiate from its surface as much energy as many it generates into its core. At this condition, the stellar substance is existing in stationary state and time derivatives from any thermodynamical functions which is characterizing for stellar substance are equal to zero:

$$\frac{dX}{dt} = 0. \tag{1.4}$$

Particularly, the time derivative of the entropy must be equal to zero in this case. I.e. conditions of an existing of each small volume of stellar substance can be considered as adiabatic one in spite of the presence of the non-equilibrium radiation. We shall use this simplification in Section VI.

The second simplification can be obtained if to suppose that a stationary star reaches the minimum of its energy after milliards years of development. (Herewith we exclude from our consideration stars with "active lifestyle". The interesting problem of the transformation of a star falls out of examination too).

The minimum condition of the star energy gives possibility to determine main parameters of equilibrium stellar substance - its density and temperature.

It is reasonable to start the development of the star theory from this point. So the problem of existing of the energy-favorable density of the stellar substance and its temperature will be considered in the first place in the next Section.

Chapter 2

The energy-favorable state of hot dense plasma

2.1 The properties of a hot dense plasma

2.1.1 A hot plasma and Boltzman's distribution

Free electrons being fermions obey the Fermi-Dirac statistic at low temperatures. At high temperatures, quantum distinctions in behavior of electron gas disappear and it is possible to consider electron gas as the ideal gas which obeys the Boltzmann's statistics. At high temperatures and high densities, all substances transform into electron-nuclear plasma. There are two tendencies in this case. At temperature much higher than the Fermi temperature $T_F = \frac{\mathcal{E}_F}{k}$ (where $\mathcal{E}_F$ is Fermi energy), the role of quantum effects is small. But their role grows with increasing of the pressure and density of an electron gas. When quantum distinctions are small, it is possible to describe the plasma electron gas as a the ideal one. The criterium of Boltzman's statistics applicability

$$T \gg \frac{\mathcal{E}_F}{k}. \tag{2.1}$$

hold true for a non-relativistic electron gas with density 10^{25} particles in cm^3 at $T \gg 10^6 K$.

At this temperatures, a plasma has energy

$$\mathcal{E} = \frac{3}{2}kTN \tag{2.2}$$

and its EOS is the ideal gas EOS:

$$P = \frac{NkT}{V} \tag{2.3}$$

But even at so high temperatures, an electron-nuclear plasma can be considered as ideal gas in the first approximation only. For more accurate description its properties, the specificity of the plasma particle interaction must be taken into account and two main corrections to ideal gas law must be introduced.

The first correction takes into account the quantum character of electrons, which obey the Pauli principle, and cannot occupy levels of energetic distribution which are already occupied by other electrons. This correction must be positive because it leads to an increased gas incompressibility.

Other correction takes into account the correlation of the screening action of charged particles inside dense plasma. It is the so-called correlational correction. Inside a dense plasma, the charged particles screen the fields of other charged particles. It leads to a decreasing of the pressure of charged particles. Accordingly, the correction for the correlation of charged particles must be negative,because it increases the compressibility of electron gas.

2.1.2 The hot plasma energy with taking into account the correction for the Fermi-statistic

The energy of the electron gas in the Boltzmann's case ($kT \gg \mathcal{E}_F$) can be calculated using the expression of the full energy of a non-relativistic Fermi-particle system [12]:

$$\mathcal{E} = \frac{2^{1/2} V m_e^{3/2}}{\pi^2 \hbar^3} \int_0^\infty \frac{\varepsilon^{3/2} d\varepsilon}{e^{(\varepsilon-\mu_e)/kT}+1}, \tag{2.4}$$

expanding it in a series. (m_e is electron mass, ε is the energy of electron and μ_e is its chemical potential).

In the Boltzmann's case, $\mu_e < 0$ and $|\mu_e/kT| \gg 1$ and the integrand at $e^{\mu_e/kT} \ll 1$ can be expanded into a series according to powers $e^{\mu_e/kT-\varepsilon/kT}$. If we introduce the notation $z = \frac{\varepsilon}{kT}$ and conserve the two first terms of the series, we obtain

$$I \equiv (kT)^{5/2} \int_0^\infty \frac{z^{3/2} dz}{e^{z-\mu_e/kT}+1} \approx$$
$$\approx (kT)^{5/2} \int_0^\infty z^{3/2} \left(e^{\frac{\mu_e}{kT}-z} - e^{2(\frac{\mu_e}{kT}-z)} + ... \right) dz \tag{2.5}$$

or

$$\frac{I}{(kT)^{5/2}} \approx e^{\frac{\mu_e}{kT}} \Gamma\left(\frac{3}{2}+1\right) - \frac{1}{2^{5/2}} e^{\frac{2\mu_e}{kT}} \Gamma\left(\frac{3}{2}+1\right) \approx$$
$$\approx \frac{3\sqrt{\pi}}{4} e^{\mu_e/kT} \left(1 - \frac{1}{2^{5/2}} e^{\mu_e/kT}\right). \tag{2.6}$$

Thus, the full energy of the hot electron gas is

$$\mathcal{E} \approx \frac{3V}{2} \frac{(kT)^{5/2}}{\sqrt{2}} \left(\frac{m_e}{\pi\hbar^2}\right)^{3/2} \left(e^{\mu_e/kT} - \frac{1}{2^{5/2}} e^{2\mu_e/kT}\right) \tag{2.7}$$

Using the definition of a chemical potential of ideal gas (of particles with spin=1/2) [12]

$$\mu_e = kTlog\left[\frac{N_e}{2V}\left(\frac{2\pi\hbar^2}{m_e kT}\right)^{3/2}\right] \tag{2.8}$$

we obtain the full energy of the hot electron gas

$$\mathcal{E}_e \approx \frac{3}{2}kTN_e\left[1+\frac{\pi^{3/2}}{4}\left(\frac{a_B e^2}{kT}\right)^{3/2} n_e\right], \tag{2.9}$$

where $a_B = \frac{\hbar^2}{m_e e^2}$ is the Bohr radius.

2.1.3 The correction for correlation of charged particles in a hot plasma

At high temperatures, the plasma particles are uniformly distributed in space. At this limit, the energy of ion-electron interaction tends to zero. Some correlation in space distribution of particles arises as the positively charged particle groups around itself preferably particles with negative charges and vice versa. It is accepted to estimate the energy of this correlation by the method developed by Debye-Hükkel for strong electrolytes [12]. The energy of a charged particle inside plasma is equal to $e\varphi$, where e is the charge of a particle, and φ is the electric potential induced by other particles on the considered particle.

This potential inside plasma is determined by the Debye law [12]:

$$\varphi(r) = \frac{e}{r}e^{-\frac{r}{r_D}} \tag{2.10}$$

where the Debye radius is

$$r_D = \left(\frac{4\pi e^2}{kT}\sum_a n_a Z_a^2\right)^{-1/2} \tag{2.11}$$

For small values of ratio $\frac{r}{r_D}$, the potential can be expanded into a series

$$\varphi(r) = \frac{e}{r} - \frac{e}{r_D} + ... \tag{2.12}$$

The following terms are converted into zero at $r \to 0$. The first term of this series is the potential of the considered particle. The second term

$$\mathcal{E} = -e^3\sqrt{\frac{\pi}{kTV}}\left(\sum_a N_a Z_a^2\right)^{3/2} \tag{2.13}$$

is a potential induced by other particles of plasma on the charge under consideration. And so the correlation energy of plasma consisting of N_e electrons and (N_e/Z) nuclei with charge Z in volume V is [12]

$$\mathcal{E}_{corr} = -e^3\sqrt{\frac{\pi n_e}{kT}}Z^{3/2}N_e \tag{2.14}$$

2.2 The energy-preferable state of a hot plasma

2.2.1 The energy-preferable density of a hot plasma

Finally, under consideration of both main corrections taking into account the inter-particle interaction, the full energy of plasma is given by

$$\mathcal{E} \approx \frac{3}{2}kTN_e\left[1 + \frac{\pi^{3/2}}{4}\left(\frac{a_B e^2}{kT}\right)^{3/2} n_e - \frac{2\pi^{1/2}}{3}\left(\frac{Z}{kT}\right)^{3/2} e^3 n_e^{1/2}\right] \tag{2.15}$$

The plasma into a star has a feature. A star generates the energy into its inner region and radiates it from the surface. At the steady state of a star, its substance must be in the equilibrium state with a minimum of its energy. The radiation is not in equilibrium of course and can be considered as a star environment. The equilibrium state of a body in an environment is related to the minimum of the function ([12]20):

$$\mathcal{E} - T_o S + P_o V, \tag{2.16}$$

where T_o and P_o are the temperature and the pressure of an environment. At taking in to account that the star radiation is going away into vacuum, the two last items can be neglected and one can obtain the equilibrium equation of a star substance as the minimum of its full energy:

$$\frac{d\mathcal{E}_{plasma}}{dn_e} = 0. \tag{2.17}$$

Now taking into account Eq.(2.15), one obtains that an equilibrium condition corresponds to the equilibrium density of the electron gas of a hot plasma

$$n_e^{equilibrium} \equiv n_\star = \frac{16}{9\pi}\frac{Z^3}{a_B^3} \approx 1.2 \cdot 10^{24} Z^3 cm^{-3}, \tag{2.18}$$

It gives the electron density $\approx 3 \cdot 10^{25} cm^{-3}$ for the equilibrium state of the hot plasma of helium.

2.2.2 The estimation of temperature of energy-preferable state of a hot stellar plasma

As the steady-state value of the density of a hot non-relativistic plasma is known, we can obtain an energy-preferable temperature of a hot non-relativistic plasma.

The virial theorem [12, 22] claims that the full energy of particles E, if they form a stable system with the Coulomb law interaction, must be equal to their kinetic energy T with a negative sign. Neglecting small corrections at a high temperature, one can write the full energy of a hot dense plasma as

$$\mathcal{E}_{plasma} = U + \frac{3}{2}kTN_e = -\frac{3}{2}kTN_e. \tag{2.19}$$

Where $U \approx -\frac{G\mathbb{M}^2}{\mathbb{R}_0}$ is the potential energy of the system, G is the gravitational constant, $\mathbb{M}$ and $\mathbb{R}_0$ are the mass and the radius of the star.

As the plasma temperature is high enough, the energy of the black radiation cannot be neglected. The full energy of the stellar plasma depending on the particle energy and the black radiation energy

$$\mathcal{E}_{total} = -\frac{3}{2}kTN_e + \frac{\pi^2}{15}\left(\frac{kT}{\hbar c}\right)^3 VkT \quad (2.20)$$

at equilibrium state must be minimal, i.e.

$$\left(\frac{\partial \mathcal{E}_{total}}{\partial T}\right)_{N,V} = 0. \quad (2.21)$$

This condition at $\frac{N_e}{V} = n_\star$ gives a possibility to estimate the temperature of the hot stellar plasma at the steady state:

$$\mathbb{T}_\star \approx Z\frac{\hbar c}{ka_B} \approx 10^7 Z\ K. \quad (2.22)$$

The last obtained estimation can raise doubts. At "terrestrial" conditions, the energy of any substance reduces to a minimum at $T \to 0$. It is caused by a positivity of a heat capacity of all of substances. But the steady-state energy of star is negative and its absolute value increases with increasing of temperature (Eq.(2.19)). It is the main property of a star as a thermodynamical object. This effect is a reflection of an influence of the gravitation on a stellar substance and is characterized by a negative effective heat capacity. The own heat capacity of a stellar substance (without gravitation) stays positive. With the increasing of the temperature, the role of the black radiation increases ($\mathcal{E}_{br} \sim T^4$). When its role dominates, the star obtains a positive heat capacity. The energy minimum corresponds to a point between these two branches.

2.2.3 Are accepted assumptions correct?

At expansion in series of the full energy of a Fermi-gas, it was supposed that the condition of applicability of Boltzmann-statistics (2.1) is valid. The substitution of obtained values of the equilibrium density $n_\star$ (Eq.(2.18)) and equilibrium temperature $\mathbb{T}_\star$ (Eq.(2.22)) shows that the ratio

$$\frac{\mathcal{E}_F(n_\star)}{k\mathbb{T}_\star} \approx 3.1Z\alpha \ll 1. \quad (2.23)$$

Where $\alpha \approx \frac{1}{137}$ is fine structure constant.

At appropriate substitution, the condition of expansion in series of the electric potential (2.12) gives

$$\frac{r}{r_D} \approx (n_\star^{1/3} r_D)^{-1} \approx \alpha^{1/2} \ll 1. \quad (2.24)$$

Thus, obtained values of steady-state parameters of plasma are in full agreement with assumptions which was made above.

Chapter 3

The gravity induced electric polarization in a dense hot plasma

3.1 Plasma cells

The existence of plasma at energetically favorable state with the constant density $n_\star$ and the constant temperature $\mathbb{T}_\star$ puts a question about equilibrium of this plasma in a gravity field. The Euler equation in commonly accepted form Eq.(1.1) disclaims a possibility to reach the equilibrium in a gravity field at a constant pressure in the substance: the gravity inevitably must induce a pressure gradient into gravitating matter. To solve this problem, it is necessary to consider the equilibrium of a dense plasma in an gravity field in detail. At zero approximation, at a very high temperature, plasma can be considered as a "jelly", where electrons and nuclei are "smeared" over a volume. At a lower temperature and a high density, when an interpartical interaction cannot be neglected, it is accepted to consider a plasma dividing in cells [15]. Nuclei are placed at centers of these cells, the rest of their volume is filled by electron gas. Its density decreases from the center of a cell to its periphery. Of course, this dividing is not freezed. Under action of heat processes, nuclei move. But having a small mass, electrons have time to trace this moving and to form a permanent electron cloud around nucleus, i.e. to form a cell. So plasma under action of a gravity must be characterized by two equilibrium conditions:

- the condition of an equilibrium of the heavy nucleus inside a plasma cell;
- the condition of an equilibrium of the electron gas, or equilibrium of cells.

3.2 The equilibrium of a nucleus inside plasma cell filled by an electron gas

At the absence of gravity, the negative charge of an electron cloud inside a cell exactly balances the positive charge of the nucleus at its center. Each cell is fully electroneutral. There is no direct interaction between nuclei.

The gravity acts on electrons and nuclei at the same time. Since the mass of nuclei is large, the gravity force applied to them is much larger than the force applied to electrons. On the another hand, as nuclei have no direct interaction, the elastic properties of plasma are depending on the electron gas reaction. Thus there is a situation, where the force applied to nuclei must be balanced by the force of the electron subsystem. The cell obtains an electric dipole moment d_s, and the plasma obtains polarization $\mathfrak{P} = n_s\ d_s$, where n_s is the density of the cell.

It is known [13], that the polarization of neighboring cells induces in the considered cell the electric field intensity

$$E_s = \frac{4\pi}{3}\mathfrak{P}, \tag{3.1}$$

and the cell obtains the energy

$$\mathcal{E}_s = \frac{d_s\ E_s}{2}. \tag{3.2}$$

The gravity force applied to the nucleus is proportional to its mass Am_p (where A is a mass number of the nucleus, m_p is the proton mass). The cell consists of Z electrons, the gravity force applied to the cell electron gas is proportional to Zm_e (where m_e is the electron mass). The difference of these forces tends to pull apart centers of positive and negative charges and to increase the dipole moment. The electric field E_s resists it. The process obtains equilibrium at the balance of the arising electric force $\nabla\mathcal{E}_s$ and the difference of gravity forces applied to the electron gas and the nucleus:

$$\nabla\left(\frac{2\pi}{3}\frac{\mathfrak{P}^2}{n_s}\right) + (Am_p - Zm_e)\mathbf{g} = 0 \tag{3.3}$$

Taking into account, that $\mathbf{g} = -\nabla\psi$, we obtain

$$\frac{2\pi}{3}\frac{\mathfrak{P}^2}{n_s} = (Am_p - Zm_e)\psi. \tag{3.4}$$

Hence,

$$\mathfrak{P}^2 = \frac{3GM_r}{2\pi r}n_e\left(\frac{A}{Z}m_p - m_e\right), \tag{3.5}$$

where ψ is the potential of the gravitational field, $n_s = \frac{n_e}{Z}$ is the density of cell (nuclei), n_e is the density of the electron gas, M_r is the mass of a star containing inside a sphere with radius r.

3.3 The equilibrium in plasma electron gas subsystem

Nonuniformly polarized matter can be represented by an electric charge distribution with density [13]

$$\widetilde{\varrho} = \frac{divE_s}{4\pi} = \frac{div\mathfrak{P}}{3}. \tag{3.6}$$

The full electric charge of cells placed inside the sphere with radius r

$$Q_r = 4\pi \int_0^r \widetilde{\varrho} r^2 dr \tag{3.7}$$

determinants the electric field intensity applied to a cell placed on a distance r from center of a star

$$\widetilde{\mathbf{E}} = \frac{Q_r}{r^2} \tag{3.8}$$

As a result, the action of a nonuniformly polarized environment can be described by the force $\widetilde{\varrho}\widetilde{E}$. This force must be taken into account in the formulating of equilibrium equation. It leads to the following form of the Euler equation:

$$\gamma\mathbf{g} + \widetilde{\varrho}\widetilde{\mathbf{E}} + \nabla P = 0 \tag{3.9}$$

Chapter 4

The internal structure of a star

It was shown above that the state with the constant density is energetically favorable for a plasma at a very high temperature. The plasma in the central region of a star can possess by this property . The calculation made below shows that the mass of central region of a star with the constant density - the star core - is equal to 1/2 of the full star mass. Its radius is approximately equal to 1/10 of radius of a star, i.e. the core with high density take approximately 1/1000 part of the full volume of a star. The other half of a stellar matter is distributed over the region placed above the core. It has a relatively small density and it could be called as a star atmosphere.

4.1 The plasma equilibrium in the star core

In this case, the equilibrium condition (Eq.(3.3)) for the energetically favorable state of plasma with the steady density $n_s = const$ is achieved at

$$\mathfrak{P} = \sqrt{G}\gamma_\star r, \tag{4.1}$$

Here the mass density is $\gamma_\star \approx \frac{A}{Z} m_p n_\star$. The polarized state of the plasma can be described by a state with an electric charge at the density

$$\widetilde{\varrho} = \frac{1}{3} div\mathfrak{P} = \sqrt{G}\gamma_\star, \tag{4.2}$$

and the electric field applied to a cell is

$$\widetilde{\mathbf{E}} = \frac{\mathbf{g}}{\sqrt{G}}. \tag{4.3}$$

As a result, the electric force applied to the cell will fully balance the gravity action

$$\gamma \mathbf{g} + \widetilde{\varrho}\widetilde{\mathbf{E}} = 0 \tag{4.4}$$

at the zero pressure gradient

$$\nabla P = 0. \tag{4.5}$$

4.2 The main parameters of a star core (in order of values)

At known density $n_\star$ of plasma into a core and its equilibrium temperature $\mathbb{T}_\star$, it is possible to estimate the mass $\mathbb{M}_\star$ of a star core and its radius $\mathbb{R}_\star$. In accordance with the virial theorem[1], the kinetic energy of particles composing the steady system must be approximately equal to its potential energy with opposite sign:

$$\frac{G\mathbb{M}_\star^2}{\mathbb{R}_\star} \approx k\mathbb{T}_\star\mathbb{N}_\star. \tag{4.6}$$

Where $\mathbb{N}_\star = \frac{4\pi}{3}\mathbb{R}_\star^3 n_\star$ is full number of particle into the star core.

With using determinations derived above (2.18) and (2.22) derived before, we obtain

$$\mathbb{M}_\star \approx \frac{\mathbb{M}_{Ch}}{(A/Z)^2} \tag{4.7}$$

where $\mathbb{M}_{Ch} = \left(\frac{\hbar c}{G m_p^2}\right)^{3/2} m_p$ is the Chandrasekhar mass.

The radius of the core is approximately equal

$$\mathbb{R}_\star \approx \left(\frac{\hbar c}{G m_p^2}\right)^{1/2} \frac{a_B}{Z(A/Z)}. \tag{4.8}$$

where A and Z are the mass and the charge number of atomic nuclei the plasma consisting of.

4.3 The equilibrium state of the plasma inside the star atmosphere

The star core is characterized by the constant mass density, the charge density, the temperature and the pressure. At a temperature typical for a star core, the plasma can be considered as ideal gas, as interactions between its particles are small in comparison with $k\mathbb{T}_\star$. In atmosphere, near surface of a star, the temperature is approximately by $3 \div 4$ orders smaller. But the plasma density is lower. Accordingly, interparticle interaction is lower too and we can continue to consider this plasma as ideal gas.

[1] Below we shell use this theorem in its more exact formulation.

In the absence of the gravitation, the equilibrium state of ideal gas in some volume comes with the pressure equalization, i.e. with the equalization of its temperature T and its density n. This equilibrium state is characterized by the equalization of the chemical potential of the gas μ (Eq.(2.8)).

4.4 The radial dependence of density and temperature of substance inside a star atmosphere

For the equilibrium system, where different parts have different temperatures, the following relation of the chemical potential of particles to its temperature holds ([12],25):

$$\frac{\mu}{kT} = const \tag{4.9}$$

As thermodynamic (statistical) part of chemical potential of monoatomic ideal gas is [12],45:

$$\mu_T = kT\ ln\left[\frac{n}{2}\left(\frac{2\pi\hbar^2}{mkT}\right)^{3/2}\right], \tag{4.10}$$

we can conclude that at the equilibrium

$$n \sim T^{3/2}. \tag{4.11}$$

In external fields the chemical potential of a gas [12]25 is equal to

$$\mu = \mu_T + \mathcal{E}^{potential} \tag{4.12}$$

where $\mathcal{E}^{potential}$ is the potential energy of particles in the external field. Therefore in addition to fulfillment of condition Eq. (4.11), in a field with Coulomb potential, the equilibrium needs a fulfillment of the condition

$$-\frac{GM_r\gamma}{rkT_r} + \frac{\mathfrak{P}_r^2}{2kT_r} = const \tag{4.13}$$

(where m is the particle mass, M_r is the mass of a star inside a sphere with radius r, $\mathfrak{P}_r$ and T_r are the polarization and the temperature on its surface. As on the core surface, the left part of Eq.(4.13) vanishes, in the atmosphere

$$M_r \sim rkT_r. \tag{4.14}$$

Supposing that a decreasing of temperature inside the atmosphere is a power function with the exponent x, its value on a radius r can be written as

$$T_r = \mathbb{T}_\star\left(\frac{\mathbb{R}_\star}{r}\right)^x \tag{4.15}$$

and in accordance with Eq.(4.11), the density

$$n_r = n_\star \left(\frac{\mathbb{R}_\star}{r}\right)^{3x/2}. \quad (4.16)$$

Setting the powers of r in the right and the left parts of the condition Eq.(4.14) equal, one can obtain $x = 4$.

Thus, at using power dependencies for the description of radial dependencies of density and temperature, we obtain

$$n_r = n_\star \left(\frac{\mathbb{R}_\star}{r}\right)^{6} \quad (4.17)$$

and

$$T_r = \mathbb{T}_\star \left(\frac{\mathbb{R}_\star}{r}\right)^{4}. \quad (4.18)$$

4.5 The mass of the star atmosphere and the full mass of a star

After integration of Eq.(4.17), we can obtain the mass of the star atmosphere

$$\mathbb{M}_A = 4\pi \int_{\mathbb{R}_\star}^{\mathbb{R}_0} (A/Z) m_p n_\star \left(\frac{\mathbb{R}_\star}{r}\right)^{6} r^2 dr = \frac{4\pi}{3}(A/Z) m_p n_\star {\mathbb{R}_\star}^3 \left[1 - \left(\frac{\mathbb{R}_\star}{\mathbb{R}_0}\right)^3\right] \quad (4.19)$$

It is equal to its core mass (to $\frac{\mathbb{R}_\star^3}{\mathbb{R}_0^3} \approx 10^{-3}$), where $\mathbb{R}_0$ is radius of a star.

Thus, the full mass of a star

$$\mathbb{M} = \mathbb{M}_A + \mathbb{M}_\star \approx 2\mathbb{M}_\star \quad (4.20)$$

Chapter 5

The virial theorem and main parameters of a star

5.1 The energy of a star

The virial theorem [12, 22] is applicable to a system of particles if they have a finite moving into a volume V. If their interaction obeys to the Coulomb's law, their potential energy $\mathcal{E}^{potential}$, their kinetic energy $\mathcal{E}^{kinetic}$ and pressure P are in the ratio:

$$2\mathcal{E}^{kinetic} + \mathcal{E}^{potential} = 3PV. \tag{5.1}$$

On the star surface, the pressure is absent and for the particle system as a whole:

$$2\mathcal{E}^{kinetic} = -\mathcal{E}^{potential} \tag{5.2}$$

and the full energy of plasma particles into a star

$$\mathcal{E}(plasma) = \mathcal{E}^{kinetic} + \mathcal{E}^{potential} = -\mathcal{E}^{kinetic}. \tag{5.3}$$

Let us calculate the separate items composing the full energy of a star.

5.1.1 The kinetic energy of plasma

The kinetic energy of plasma into a core:

$$\mathcal{E}_\star^{kinitic} = \frac{3}{2}k\mathbb{T}_\star\mathbb{N}_\star. \tag{5.4}$$

The kinetic energy of atmosphere:

$$\mathcal{E}_a^{kinetic} = 4\pi\int_{\mathbb{R}_\star}^{\mathbb{R}_0}\frac{3}{2}k\mathbb{T}_\star n_\star\left(\frac{\mathbb{R}_\star}{r}\right)^{10}r^2dr \approx \frac{3}{7}\left(\frac{3}{2}k\mathbb{T}_\star\mathbb{N}_\star\right) \tag{5.5}$$

The total kinetic energy of plasma particles

$$\mathcal{E}^{kinetic} = \mathcal{E}_{\star}^{kinetic} + \mathcal{E}_{a}^{kinetic} = \frac{15}{7} k\mathbb{T}_{\star}\mathbb{N}_{\star} \tag{5.6}$$

5.1.2 The potential energy of star plasma

Inside a star core, the gravity force is balanced by the force of electric nature. Correspondingly, the energy of electric polarization can be considered as balanced by the gravitational energy of plasma. As a result, the potential energy of a core can be considered as equal to zero.

In a star atmosphere, this balance is absent.

The gravitational energy of an atmosphere

$$\mathcal{E}_a^G = -4\pi G\mathbb{M}_{\star}\frac{A}{Z}m_p n_{\star}\int_{\mathbb{R}_{\star}}^{\mathbb{R}_0}\frac{1}{2}\left[2-\left(\frac{\mathbb{R}_{\star}}{r}\right)^3\right]\left(\frac{\mathbb{R}_{\star}}{r}\right)^6 rdr \tag{5.7}$$

or

$$\mathcal{E}_a^G = \frac{3}{2}\left(\frac{1}{7}-\frac{1}{2}\right)\frac{G\mathbb{M}_{\star}^2}{\mathbb{R}_{\star}} = -\frac{15}{28}\frac{G\mathbb{M}_{\star}^2}{\mathbb{R}_{\star}} \tag{5.8}$$

The electric energy of atmosphere is

$$\mathcal{E}_a^E = -4\pi\int_{\mathbb{R}_{\star}}^{R_0}\frac{1}{2}\varrho\varphi r^2 dr, \tag{5.9}$$

where

$$\widetilde{\varrho} = \frac{1}{3r^2}\frac{d\mathfrak{P}r^2}{dr} \tag{5.10}$$

and

$$\widetilde{\varphi} = \frac{4\pi}{3}\mathfrak{P}r. \tag{5.11}$$

The electric energy:

$$\mathcal{E}_a^E = -\frac{3}{28}\frac{G\mathbb{M}_{\star}^2}{\mathbb{R}_{\star}}, \tag{5.12}$$

and total potential energy of atmosphere:

$$\mathcal{E}_a^{potential} = \mathcal{E}_a^G + \mathcal{E}_a^E = -\frac{9}{14}\frac{G\mathbb{M}_{\star}^2}{\mathbb{R}_{\star}}. \tag{5.13}$$

The equilibrium in a star depends both on plasma energy and energy of radiation.

5.2 The temperature of a star core

5.2.1 The energy of the black radiation

The energy of black radiation inside a star core is

$$\mathcal{E}_{\star}(br) = \frac{\pi^2}{15}k\mathbb{T}_{\star}\left(\frac{k\mathbb{T}_{\star}}{\hbar c}\right)^3\mathbb{V}_{\star}. \tag{5.14}$$

The energy of black radiation inside a star atmosphere is

$$\mathcal{E}_a(br) = 4\pi \int_{R_\star}^{R_0} \frac{\pi^2}{15} k\mathbb{T}_\star \left(\frac{k\mathbb{T}_\star}{\hbar c}\right)^3 \left(\frac{\mathbb{R}_\star}{r}\right)^{16} r^2 dr = \frac{3}{13}\frac{\pi^2}{15} k\mathbb{T}_\star \left(\frac{k\mathbb{T}_\star}{\hbar c}\right)^3 \mathbb{V}_\star. \tag{5.15}$$

The total energy of black radiation inside a star is

$$\mathcal{E}(br) = \mathcal{E}_\star(br) + \mathcal{E}_a(br) = \frac{16}{13}\frac{\pi^2}{15} kT_\star \left(\frac{kT_\star}{\hbar c}\right)^3 V_\star = 1.23\frac{\pi^2}{15} kT_\star \left(\frac{kT_\star}{\hbar c}\right)^3 V_\star \tag{5.16}$$

5.2.2 The full energy of a star

In accordance with (5.3), the full energy of a star

$$\mathcal{E}^{star} = -\mathcal{E}^{kinetic} + \mathcal{E}(br) \tag{5.17}$$

i.e.

$$\mathcal{E}^{star} = -\frac{15}{7} k\mathbb{T}_\star \mathbb{N}_\star + \frac{16}{13}\frac{\pi^2}{15} k\mathbb{T}_\star \left(\frac{k\mathbb{T}_\star}{\hbar c}\right)^3 \mathbb{V}_\star. \tag{5.18}$$

The steady state of a star is determined by a minimum of its full energy:

$$\left(\frac{d\mathcal{E}^{star}}{d\mathbb{T}_\star}\right)_{\mathbb{N}=const,\mathbb{V}=const} = 0, \tag{5.19}$$

it corresponds to the condition:

$$-\frac{15}{7}\mathbb{N}_\star + \frac{64\pi^2}{13\cdot 15}\left(\frac{k\mathbb{T}_\star}{\hbar c}\right)^3 \mathbb{V}_\star = 0. \tag{5.20}$$

Together with Eq.(2.18) it defines the equilibrium temperature of a star core:

$$\mathbb{T}_\star = \left(\frac{25\cdot 13}{28\pi^4}\right)^{1/3} \left(\frac{\hbar c}{k a_B}\right) Z \approx Z \cdot 2.13 \cdot 10^7 K \tag{5.21}$$

5.3 Main star parameters

5.3.1 The star mass

The virial theorem connect kinetic energy of a system with its potential energy. In accordance with Eqs.(5.13) and (5.6)

$$\frac{9}{14}\frac{G\mathbb{M}_\star^2}{\mathbb{R}_\star} = \frac{30}{7} k\mathbb{T}_\star \mathbb{N}_\star. \tag{5.22}$$

Introducing the non-dimensional parameter

$$\eta = \frac{G\mathbb{M}_\star \frac{A}{Z} m_p}{\mathbb{R}_\star k\mathbb{T}_\star}, \tag{5.23}$$

we obtain

$$\eta = \frac{20}{3} = 6.67, \tag{5.24}$$

and at taking into account Eqs.(2.18) and (5.21), the core mass is

$$\mathbb{M}_\star = \left[\frac{20}{3}\left(\frac{25\cdot 13}{28}\right)^{1/3}\frac{3}{4\cdot 3.14}\right]^{3/2}\frac{\mathbb{M}_{Ch}}{\left(\frac{A}{Z}\right)^2} = 6.84\frac{\mathbb{M}_{Ch}}{\left(\frac{A}{Z}\right)^2} \tag{5.25}$$

The obtained equation plays a very important role, because together with Eq.(4.20), it gives a possibility to predict the total mass of a star:

$$\mathbb{M} = 2\mathbb{M}_\star = \frac{13.68\mathbb{M}_{Ch}}{\left(\frac{A}{Z}\right)^2} \approx \frac{25.34\mathbb{M}_\odot}{\left(\frac{A}{Z}\right)^2}. \tag{5.26}$$

The comparison of obtained prediction Eq.(5.26) with measuring data gives a method to check our theory. Although there is no way to determine chemical composition of cores of far stars, some predictions can be made in this way. At first, there must be no stars which masses exceed the mass of the Sun by more than one and a half orders, because it accords to limiting mass of stars consisting from hydrogen with $A/Z = 1$. Secondly, the action of a specific mechanism (see. Sec.10) can make neutron-excess nuclei stable, but it don't give a base to suppose that stars with $A/Z > 10$ (and with mass in hundred times less than hydrogen stars) can exist. Thus, the theory predicts that the whole mass spectrum must be placed in the interval from 0.25 up to approximately 25 solar masses. These predications are verified by measurements quite exactly. The mass distribution of binary stars[1] is shown in Fig.5.1 [10].

The mass spectrum of close binary stars[2] is shown in Fig.5.2.

The very important element of the direct and clear confirmation of the theory is obviously visible on these figures - both spectra is dropped near the value $A/Z = 1$.

Beside it, one can easy see, that the mass spectrum of binary stars (Fig.(5.1)) consists of series of well-isolated lines which are representing the stars with integer values of ratios $A/Z = 3, 4, 5...$, corresponding hydrogen-3,4,5 ... or helium-6,8,10 ... (also line with the half-integer ratio $A/Z = 3/2$, corresponding, probably, to helium-3, Be-6, C-9...). The existence of stable stars with ratios $A/Z \geq 3$ raises questions. It is generally assumed that stars are composed of hydrogen-1, deuterium, helium-4 and other heavier elements with $A/Z \approx 2$. Nuclei with $A/Z \geq 3$ are the neutron-excess and so short-lived, that they can not build a long-lived stars. Neutron-excess nuclei can become stable under the action of mechanism of neutronization, which is acting inside the dwarfs. It is accepted to think that this mechanism must not work into

[1] The use of these data is caused by the fact that only the measurement of parameters of binary star rotation gives a possibility to determine their masses with satisfactory accuracy.

[2] The data of these measurements were obtained in different observatories of the world. The last time the summary table with these data was gathered by Khaliulilin Kh.F. (Sternberg Astronomical Institute) [11] in his dissertation (in Russian) which has unfortunately has a restricted access. With his consent and for readers convenience, we place that table in Appendix.

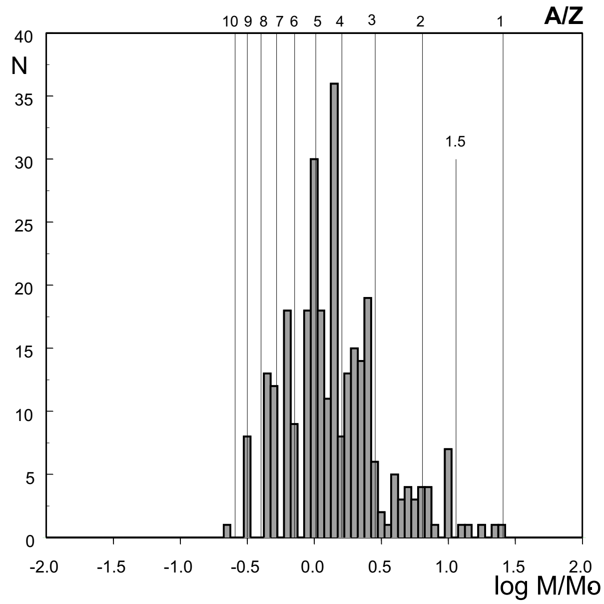

Figure 5.1: The mass distribution of binary stars [10]. On abscissa, the logarithm of the star mass over the Sun mass is shown. Solid lines mark masses, which agree with selected values of A/Z from Eq.(5.26).

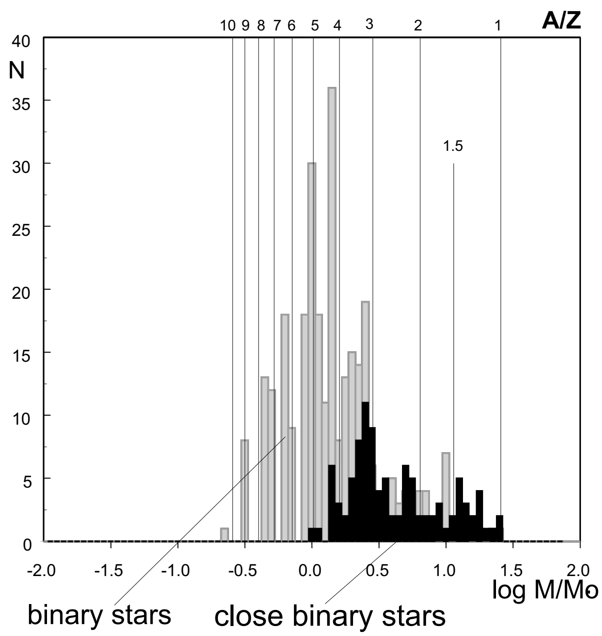

Figure 5.2: The mass distribution of close binary stars [11]. On abscissa, the logarithm of the star mass over the Sun mass is shown. Solid lines mark masses, which agree with selected values of A/Z from Eq.(5.26). The binary star spectrum is shown for comparison.

the stars. The consideration of the effecting of the electron gas of a dense plasma on the nucleus is described in chapter (10). The calculations of Ch.(10) show that the electron gas of dense plasma should also lead to the neutronization mechanism and to the stabilization of the neutron-excess nuclei. This explains the existence of a stable of stars, where the plasma consists of nuclei with $A/Z \geq 3$.

Beside it, at considering of Fig.(5.1), the question is arising: why there are so few stars, which are composed by very stable nuclei of helium-4? At the same time, there are many stars with $A/Z = 4$, i.e. consisting apparently of a hydrogen-4, as well as stars with $A/Z = 3/2$, which hypothetically could be composed by another isotope of helium - helium-3. This equation is discussed in Ch.(10).

Beside it, it is important to note, that according to Eq.(5.26) the Sun must consist from a substance with $A/Z = 5$. This conclusion is in a good agreement with results of consideration of solar oscillations (Chapter 9).

5.3.2 Radii of stars

Using Eq.(2.18) and Eq.(5.25), we can determine the star core radius:

$$\mathbb{R}_\star = 1.42 \frac{a_B}{Z(A/Z)} \left(\frac{\hbar c}{G m_p^2}\right)^{1/2} \approx \frac{9.79 \cdot 10^{10}}{Z(A/Z)} cm. \tag{5.27}$$

The temperature near the star surface is relatively small. It is approximately by 3 orders smaller than it is inside the core. Because of it at calculation of surface parameters, we must take into consideration effects of this order, i.e. it is necessary to take into account the gravity action on the electron gas. At that it is convenient to consider the plasma cell as some neutral quasi-atom (like the Thomas-Fermi atom). Its electron shell is formed by a cloud of free electrons.

Each such quasi-atom is retained on the star surface by its negative potential energy

$$(\mathcal{E}_{gravitational} + \mathcal{E}_{electric}) < 0. \tag{5.28}$$

The electron cloud of the cell is placed in the volume $\delta V = \frac{4\pi}{3} r_s^3$, (where $r_s \approx \left(\frac{Z}{n_e}\right)^{1/3}$) under pressure P_e. The evaporation of plasma cell releases energy $\mathcal{E}_{PV} = P_e V_s$, and the balance equation takes the form:

$$\mathcal{E}_{gravitational} + \mathcal{E}_{electric} + \mathcal{E}_{PV} = 0. \tag{5.29}$$

In cold plasma, the electron cloud of the cell has energy $\mathcal{E}_{PV} \approx e^2 {n_e}^{1/3}$. in very hot plasma at $kT \gg \frac{Z^2 e^2}{r_s}$, this energy is equal to $\mathcal{E}_{PV} = \frac{3}{2} ZkT$. On the star surface these energies are approximately equal:

$$\frac{k\mathbb{T}_0}{e^2 n_e^{1/3}} \approx \frac{1}{\alpha} \left(\frac{\mathbb{R}_0}{\mathbb{R}_\star}\right)^2 \approx 1. \tag{5.30}$$

One can show it easily, that in this case

$$\mathcal{E}_{PV} \approx 2Z\sqrt{\frac{3}{2}kT \cdot e^2 {n_e}^{1/3}}. \tag{5.31}$$

And if to take into account Eqs.(4.17)-(4.18), we obtain

$$\mathcal{E}_{PV} \approx 1.5Zk\mathbb{T}_\star \left(\frac{\mathbb{R}_\star}{\mathbb{R}_0}\right)^3 \sqrt{\alpha\pi} \tag{5.32}$$

The energy of interaction of a nucleus with its electron cloud does not change at evaporation of the cell and it can be neglected. Thus, for the surface

$$\mathcal{E}_{electric} = \frac{2\pi\mathfrak{P}^2}{3n_s} = \frac{2G\mathbb{M}_\star}{\mathbb{R}_0}\left(Am_p - Zm_e\right). \tag{5.33}$$

The gravitational energy of the cell on the surface

$$\mathcal{E}_{gravitational} = -\frac{2G\mathbb{M}_\star}{\mathbb{R}_0}\left(Am_p + Zm_e\right). \tag{5.34}$$

Thus, the balance condition Eq.(5.29) on the star surface obtains the form

$$-\frac{4G\mathbb{M}_\star Zm_e}{\mathbb{R}_0} + 1.5Zk\mathbb{T}_\star \left(\frac{\mathbb{R}_\star}{\mathbb{R}_0}\right)^3 \sqrt{\alpha\pi} = 0. \tag{5.35}$$

5.3.3 The $\mathbb{R}_\star/\mathbb{R}_0$ ratio and $\mathbb{R}_0$

With account of Eq.(4.18) and Eqs.(5.24)-(5.23), we can write

$$\frac{\mathbb{R}_0}{\mathbb{R}_\star} = \left(\frac{\sqrt{\alpha\pi}}{2\eta}\frac{\frac{A}{Z}m_p}{m_e}\right)^{1/2} \approx 4.56\sqrt{\frac{A}{Z}} \tag{5.36}$$

As the star core radius is known Eq.(5.27), we can obtain the star surface radius:

$$\mathbb{R}_0 \approx \frac{4.46 \cdot 10^{11}}{Z(A/Z)^{1/2}} cm. \tag{5.37}$$

5.3.4 The temperature of a star surface

At known Eq.(4.18) and Eq.(5.21), we can calculate the temperature of external surface of a star

$$\mathbb{T}_0 = \mathbb{T}_\star \left(\frac{\mathbb{R}_\star}{\mathbb{R}_0}\right)^4 \approx 4.92 \cdot 10^5 \frac{Z}{(A/Z)^2} \tag{5.38}$$

5.3.5 The comparison with measuring data

The mass spectrum (Fig.5.1) shows that the Sun consists basically from plasma with $A/Z = 5$. The radius of the Sun and its surface temperature are functions of Z too. This values calculated at A/Z=5 and differen Z are shown in Table (5.3.3)

Table (5.3.3)

Z	$\mathbb{R}_\odot, cm$ (calculated (5.37))	$\mathbb{T}_\odot$,K (calculated (5.38))
1	$2.0 \cdot 10^{11}$	1961
2	$1.0 \cdot 10^{11}$	3923
3	$6.65 \cdot 10^{10}$	5885
4	$5.0 \cdot 10^{10}$	7845

One can see that these calculated data have a satisfactory agreement the measured radius of the Sun

$$\mathbb{R}_\odot = 6.96 \cdot 10^{10} cm \tag{5.39}$$

and the measured surface temperature

$$\mathbb{T}_\odot = 5850\ K \tag{5.40}$$

at $Z = 3$.

The calculation shows that the mass of core of the Sun

$$\mathbb{M}_\star(Z = 3, A/Z = 5) \approx 9.68 \cdot 10^{32}\ g \tag{5.41}$$

i.t. almost exactly equals to one half of full mass of the Sun

$$\frac{\mathbb{M}_\star(Z = 3, A/Z = 5)}{\mathbb{M}_\odot} \approx 0.486 \tag{5.42}$$

in full agreement with Eq.(4.20).

In addition to obtained determinations of the mass of a star Eq.(5.26), its temperature Eq.(5.38) and its radius Eq.(5.37) give possibility to check the calculation, if we compare these results with measuring data. Really, dependencies measured by astronomers can be described by functions:

$$\mathbb{M} = \frac{Const_1}{(A/Z)^2}, \tag{5.43}$$

$$\mathbb{R}_0 = \frac{Const_2}{Z(A/Z)^{1/2}}, \tag{5.44}$$

$$\mathbb{T}_0 = \frac{Const_3 Z}{(A/Z)^2}. \tag{5.45}$$

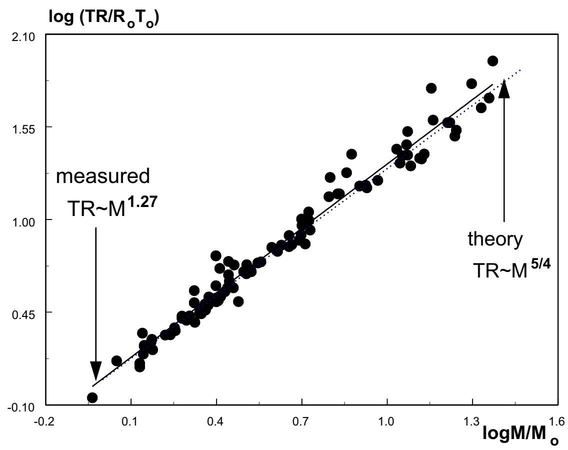

Figure 5.3: The relation between main parameters of stars (Eq.(5.46)) and corresponding data of astronomical measurements for close binary stars [11] are shown.

If to combine they in the way, to exclude unknown parameter Z, one can obtain relation:

$$\mathbb{T}_0\mathbb{R}_0 = Const\,\mathbb{M}^{5/4}, \tag{5.46}$$

Its accuracy can by checked. For this checking, let us use the measuring data of parameters of masses, temperatures and radii of close binary stars [11]. The results of these measurements are shown in Fig.(5.3), where the dependence according to Eq.(5.46). It is not difficult to see that these data are well described by the calculated dependence. It speaks about successfulness of our consideration.

If main parameters of the star are expressed through corresponding solar values $\tau \equiv \frac{\mathbb{T}_0}{\mathbb{T}_\odot}$,$\rho \equiv \frac{\mathbb{R}_0}{\mathbb{R}_\odot}$ and $\mu \equiv \frac{\mathbb{M}}{\mathbb{M}_\odot}$, that Eq.(5.46) can be rewritten as

$$\frac{\tau\rho}{\mu^{5/4}} = 1. \tag{5.47}$$

Numerical values of relations $\frac{\tau\rho}{\mu^{5/4}}$ for close binary stars [11] are shown in the last column of the Table(6.2)(at the end Chapter (6)).

Chapter 6

The thermodynamic relations of intra-stellar plasma

6.1 The thermodynamic relation of star atmosphere parameters

Hot stars steadily generate energy and radiate it from their surfaces. This is non-equilibrium radiation in relation to a star. But it may be a stationary radiation for a star in steady state. Under this condition, the star substance can be considered as an equilibrium. This condition can be considered as quasi-adiabatic, because the interchange of energy between both subsystems - radiation and substance - is stationary and it does not result in a change of entropy of substance. Therefore at consideration of state of a star atmosphere, one can base it on equilibrium conditions of hot plasma and the ideal gas law for adiabatic condition can be used for it in the first approximation.

It is known, that thermodynamics can help to establish correlation between steady-state parameters of a system. Usually, the thermodynamics considers systems at an equilibrium state with constant temperature, constant particle density and constant pressure over all system. The characteristic feature of the considered system is the existence of equilibrium at the absence of a constant temperature and particle density over atmosphere of a star. To solve this problem, one can introduce averaged pressure

$$\widehat{P} \approx \frac{G\mathbb{M}^2}{\mathbb{R}_0^4}, \tag{6.1}$$

averaged temperature

$$\widehat{T} = \frac{\int_{\mathbb{V}} T dV}{V} \sim \mathbb{T}_0 \left(\frac{\mathbb{R}_0}{\mathbb{R}_\star}\right) \tag{6.2}$$

and averaged particle density

$$\widehat{n} \approx \frac{\mathbb{N}_A}{\mathbb{R}_0^3} \tag{6.3}$$

After it by means of thermodynamical methods, one can find relation between parameters of a star.

6.1.1 The c_P/c_V ratio

At a movement of particles according to the theorem of the equidistribution, the energy $kT/2$ falls at each degree of freedom. It gives the heat capacity $c_v = 3/2k$.

According to the virial theorem [12, 22], the full energy of a star should be equal to its kinetic energy (with opposite sign)(Eq.(5.3)), so as full energy related to one particle

$$\mathcal{E} = -\frac{3}{2}kT \tag{6.4}$$

In this case the heat capacity at constant volume (per particle over Boltzman's constant k) by definition is

$$c_V = \left(\frac{dE}{dT}\right)_V = -\frac{3}{2} \tag{6.5}$$

The negative heat capacity of stellar substance is not surprising. It is a known fact and it is discussed in Landau-Lifshitz course [12]. The own heat capacity of each particle of star substance is positive. One obtains the negative heat capacity at taking into account the gravitational interaction between particles.

By definition the heat capacity of an ideal gas particle at permanent pressure [12] is

$$c_P = \left(\frac{dW}{dT}\right)_P, \tag{6.6}$$

where W is enthalpy of a gas.

As for the ideal gas [12]

$$W - \mathcal{E} = NkT, \tag{6.7}$$

and the difference between c_P and c_V

$$c_P - c_V = 1. \tag{6.8}$$

Thus in the case considered, we have

$$c_P = -\frac{1}{2}. \tag{6.9}$$

Supposing that conditions are close to adiabatic ones, we can use the equation of the Poisson's adiabat.

6.1.2 The Poisson's adiabat

The thermodynamical potential of a system consisting of N molecules of ideal gas at temperature T and pressure P can be written as [12]:

$$\Phi = const \cdot N + NTlnP - Nc_P TlnT. \qquad (6.10)$$

The entropy of this system

$$S = const \cdot N - NlnP + Nc_P lnT. \qquad (6.11)$$

As at adiabatic process, the entropy remains constant

$$-NTlnP + Nc_P TlnT = const, \qquad (6.12)$$

we can write the equation for relation of averaged pressure in a system with its volume (The Poisson's adiabat) [12]:

$$\widehat{P}V^{\widetilde{\gamma}} = const, \qquad (6.13)$$

where $\widetilde{\gamma} = \frac{c_P}{c_V}$ is the exponent of adiabatic constant. In considered case taking into account of Eqs.(6.6) and (6.5), we obtain

$$\widetilde{\gamma} = \frac{c_P}{c_V} = \frac{1}{3}. \qquad (6.14)$$

As $V^{1/3} \sim \mathbb{R}_0$, we have for equilibrium condition

$$\widehat{P}\mathbb{R}_0 = const. \qquad (6.15)$$

6.2 The mass-radius ratio

Using Eq.(6.1) from Eq.(6.15), we use the equation for dependence of masses of stars on their radii:

$$\frac{\mathbb{M}^2}{\mathbb{R}_0^3} = const \qquad (6.16)$$

This equation shows the existence of internal constraint of chemical parameters of equilibrium state of a star. Indeed, the substitution of obtained determinations Eq.(5.37) and (5.38)) into Eq.(6.16) gives:

$$Z \sim (A/Z)^{5/6} \qquad (6.17)$$

Simultaneously the observational data of masses, of radii and their temperatures was obtained by astronomers for close binary stars [11]. The dependence of radii of these stars over these masses is shown in Fig.6.1 on double logarithmic scale. The solid line shows the result of fitting of measurement data $\mathbb{R}_0 \sim \mathbb{M}^{0.68}$. It is close to theoretical dependence $\mathbb{R}_0 \sim \mathbb{M}^{2/3}$ (Eq.6.16) which is shown by dotted line.

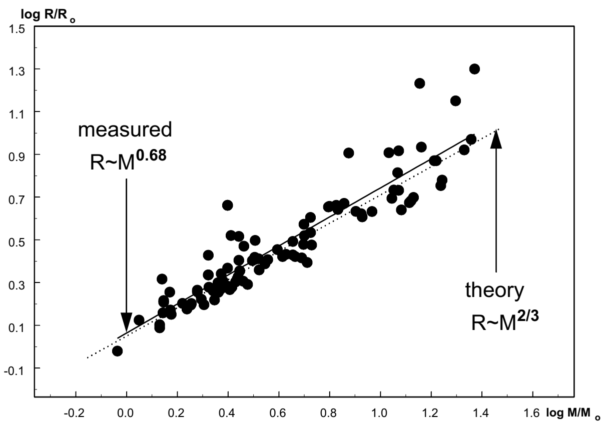

Figure 6.1: The dependence of radii of stars over the star mass [11]. Here the radius of stars is normalized to the sunny radius, the stars masses are normalized to the mass of the Sum. The data are shown on double logarithmic scale. The solid line shows the result of fitting of measurement data $\mathbb{R}_0 \sim \mathbb{M}^{0.68}$. The theoretical dependence $\mathbb{R}_0 \sim \mathbb{M}^{2/3}$ (6.16) is shown by the dotted line.

If parameters of the star are expressed through corresponding solar values $\rho \equiv \frac{\mathbb{R}_0}{\mathbb{R}_\odot}$ and $\mu \equiv \frac{\mathbb{M}}{\mathbb{M}_\odot}$, that Eq.(6.16) can be rewritten as

$$\frac{\rho}{\mu^{2/3}} = 1. \tag{6.18}$$

Numerical values of relations $\frac{\rho}{\mu^{2/3}}$ for close binary stars [11] are shown in the Table(6.2).

The Table(6.2). The relations of main stellar parameters

N	Star		$\mu \equiv \frac{\mathbb{M}}{\mathbb{M}_{\odot}}$	$\rho \equiv \frac{\mathbb{R}_0}{\mathbb{R}_{\odot}}$	$\tau \equiv \frac{\mathbb{T}_0}{\mathbb{T}_{\odot}}$	$\frac{\rho}{\mu^{2/3}}$	$\frac{\tau}{\mu^{7/12}}$	$\frac{\rho\tau}{\mu^{5/4}}$
1	BW Aqr	1	1.48	1.803	1.043	1.38	0.83	1.15
		2	1.38	2.075	1.026	1.67	0.85	1.42
2	V 889 Aql	1	2.4	2.028	1.692	1.13	1.01	1.15
		2	2.2	1.826	1.607	1.08	1.01	1.09
3	V 539 Ara	1	6.24	4.512	3.043	1.33	1.04	1.39
		2	5.31	4.512	3.043	1.12	1.09	1.23
4	AS Cam	1	3.31	2.58	1.966	1.16	0.98	1.13
		2	2.51	1.912	1.709	1.03	1.0	1.03
5	EM Car	1	22.8	9.35	5.658	1.16	0.91	1.06
		2	21.4	8.348	5.538	1.08	0.93	1.00
6	GL Car	1	13.5	4.998	5.538	0.88	1.08	0.95
		2	13	4.726	4.923	0.85	1.1	0.94
7	QX Car	1	9.27	4.292	4	0.97	1.09	1.06
		2	8.48	4.054	3.829	0.975	1.1	1.07
8	AR Cas	1	6.7	4.591	3.111	1.29	1.02	1.32
		2	1.9	1.808	1.487	1.18	1.02	1.21
9	IT Cas	1	1.4	1.616	1.102	1.29	0.91	1.17
		2	1.4	1.644	1.094	1.31	0.90	1.18
10	OX Cas	1	7.2	4.69	4.068	1.25	1.29	1.62
		2	6.3	4.54	3.93	1.33	1.34	1.79
11	PV Cas	1	2.79	2.264	1.914	1.14	1.05	1.20
		2	2.79	2.264	2.769	1.14	1.05	1.20
12	KT Cen	1	5.3	4.028	2.769	1.32	1.05	1.39
		2	5	3.745	2.701	1.28	1.06	1.35

The Table(6.2)(continuation).

N	Star	n	$\mu \equiv \frac{\mathbb{M}}{\mathbb{M}_\odot}$	$\rho \equiv \frac{\mathbb{R}_0}{\mathbb{R}_\odot}$	$\tau \equiv \frac{\mathbb{T}_0}{\mathbb{T}_\odot}$	$\frac{\rho}{\mu^{2/3}}$	$\frac{\tau}{\mu^{7/12}}$	$\frac{\rho\tau}{\mu^{5/4}}$
13	V 346 Cen	1	11.8	8.26	4.05	1.59	0.96	1.53
		2	8.4	4.19	3.83	1.01	1.11	1.12
14	CW Cep	1	11.8	8.263	4.051	1.04	1.06	1.11
		2	11.1	4.954	4.393	1.0	1.08	1.07
15	EK Cep	1	2.02	1.574	1.709	0.98	1.13	1.12
		2	1.12	1.332	1.094	1.23	1.02	1.26
16	α Cr B	1	2.58	3.314	1.555	1.76	0.89	1.57
		2	0.92	0.955	0.923	1.01	0.97	0.98
17	Y Cyg	1	17.5	6.022	5.66	0.89	1.06	0.95
		2	17.3	5.68	5.54	0.85	1.05	0.89
18	Y 380 Cyg	1	14.3	17.08	3.54	2.89	0.75	2.17
		2	8	4.3	3.69	1.07	1.1	1.18
19	V 453 Cyg	1	14.5	8.607	4.55	1.45	0.95	1.38
		2	11.3	5.41	4.44	1.07	1.08	1.16
20	V 477 Cyg	1	1.79	1.567	1.46	1.06	1.04	1.11
		2	1.35	1.27	1.11	1.04	0.93	0.97
21	V 478 Cyg	1	16.3	7.42	5.09	1.15	1.0	1.15
		2	16.6	7.42	5.09	1.14	0.99	1.13
22	V 541 Cyg	1	2.69	2.013	1.86	1.04	1.05	1.09
		2	2.6	1.9	1.85	1.0	1.6	1.06
23	V 1143 Cyg	1	1.39	1.44	1.11	1.16	0.92	0.92
		2	1.35	1.23	1.09	1.0	0.91	0.92
24	V 1765 Cyg	1	23.5	19.96	4.39	2.43	0.67	1.69
		2	11.7	6.52	4.29	1.26	1.02	1.29

The Table(6.2)(continuation).

N	Star	n	$\mu \equiv \frac{\mathbb{M}}{\mathbb{M}_\odot}$	$\rho \equiv \frac{\mathbb{R}_0}{\mathbb{R}_\odot}$	$\tau \equiv \frac{\mathbb{T}_0}{\mathbb{T}_\odot}$	$\frac{\rho}{\mu^{2/3}}$	$\frac{\tau}{\mu^{7/12}}$	$\frac{\rho\tau}{\mu^{5/4}}$
25	DI Her	1	5.15	2.48	2.91	0.83	1.12	0.93
		2	4.52	2.69	2.58	0.98	1.07	1.05
26	HS Her	1	4.25	2.71	2.61	1.03	1.12	1.16
		2	1.49	1.48	1.32	1.14	1.04	1.19
27	CO Lac	1	3.13	2.53	1.95	1.18	1.00	1.12
		2	2.75	2.13	1.86	1.08	1.01	1.09
28	GG Lup	1	6.24	4.12	2.64	1.03	1.08	1.11
		2	2.51	1.92	1.79	1.04	1.05	1.09
29	RU Mon	1	3.6	2.55	2.20	1.09	1.04	1.14
		2	3.33	2.29	2.15	1.03	1.07	1.10
30	GN Nor	1	2.5	4.59	1.33	2.49	0.78	1.95
		2	2.5	4.59	1.33	2.49	0.78	1.95
31	U Oph	1	5.02	3.31	2.80	1.13	1.09	1.23
		2	4.52	3.11	2.60	1.14	1.08	1.23
32	V 451 Oph	1	2.77	2.54	1.86	1.29	1.03	1.32
		2	2.35	1.86	1.67	1.05	1.02	1.07
33	β Ori	1	19.8	14.16	4.55	1.93	0.80	1.54
		2	7.5	8.07	3.04	2.11	0.94	1.98
34	FT Ori	1	2.5	1.89	1.81	1.03	1.06	1.09
		2	2.3	1.80	1.62	1.03	1.0	1.03
35	AG Per	1	5.36	3.0	2.91	0.98	1.09	1.06
		2	4.9	2.61	2.91	0.90	1.15	1.04
36	IQ Per	1	3.51	2.44	2.27	1.06	1.09	1.16
		2	1.73	1.50	2.27	1.04	1.00	1.05

The Table(6.2)(continuation).

N	Star	n	$\mu \equiv \frac{\mathbb{M}}{\mathbb{M}_{\odot}}$	$\rho \equiv \frac{\mathbb{R}_0}{\mathbb{R}_{\odot}}$	$\tau \equiv \frac{\mathbb{T}_0}{\mathbb{T}_{\odot}}$	$\frac{\rho}{\mu^{2/3}}$	$\frac{\tau}{\mu^{7/12}}$	$\frac{\rho\tau}{\mu^{5/4}}$
37	ς Phe	1	3.93	2.85	2.41	1.14	1.08	1.24
		2	2.55	1.85	1.79	0.99	1.04	1.03
38	KX Pup	1	2.5	2.33	1.74	1.27	1.02	1.29
		2	1.8	1.59	1.38	1.08	0.98	1.06
39	NO Pup	1	2.88	2.03	1.95	1.00	1.05	1.05
		2	1.5	1.42	1.20	1.08	0.94	1.02
40	VV Pyx	1	2.1	2.17	1.49	1.32	0.96	1.27
		2	2.1	2.17	1.49	1.32	0.96	1.27
41	YY Sgr	1	2.36	2.20	1.59	1.24	0.96	1.19
		2	2.29	1.99	1.59	1.15	0.98	1.12
42	V 523 Sgr	1	2.1	2.67	1.42	1.63	0.92	1.50
		2	1.9	1.84	1.42	1.20	0.98	1.17
43	V 526 Sgr	1	2.11	1.9	1.30	1.15	0.84	0.97
		2	1.66	1.60	1.30	1.14	0.97	1.10
44	V 1647 Sgr	1	2.19	1.83	1.52	1.09	0.96	1.05
		2	1.97	1.67	4.44	1.06	1.02	1.09
45	V 2283 Sgr	1	3.0	1.96	1.67	0.94	0.88	0.83
		2	2.22	1.66	1.67	0.97	1.05	1.02
46	V 760 Sco	1	4.98	3.02	2.70	1.03	1.06	1.09
		2	4.62	2.64	2.70	0.95	1.11	1.05
47	AO Vel	1	3.2	2.62	1.83	1.21	0.93	1.12
		2	2.9	2.95	1.83	1.45	0.98	1.43
48	EO Vel	1	3.21	3.14	1.73	1.44	0.87	1.26
		2	2.77	3.28	1.73	1.66	0.95	1.58

The Table(6.2)(continuation).

N	Star	n	$\mu \equiv \frac{\mathbb{M}}{\mathbb{M}_\odot}$	$\rho \equiv \frac{\mathbb{R}_0}{\mathbb{R}_\odot}$	$\tau \equiv \frac{\mathbb{T}_0}{\mathbb{T}_\odot}$	$\frac{\rho}{\mu^{2/3}}$	$\frac{\tau}{\mu^{7/12}}$	$\frac{\rho\tau}{\mu^{5/4}}$
49	α Vir	1	10.8	6.10	3.25	1.66	0.81	1.34
		2	6.8	4.39	3.25	1.22	1.06	1.30
50	DR Vul	1	13.2	4.81	4.79	0.83	1.06	0.91
		2	12.1	4.37	4.79	0.83	1.12	0.93

6.3 The mass-temperature and mass-luminosity relations

Taking into account Eqs.(4.18), (2.22) and (4.8) one can obtain the relation between surface temperature and the radius of a star

$$\mathbb{T}_0 \sim \mathbb{R}_0^{7/8}, \tag{6.19}$$

or accounting for (6.16)

$$\mathbb{T}_0 \sim \mathbb{M}^{7/12} \tag{6.20}$$

The dependence of the temperature on the star surface over the star mass of close binary stars [11] is shown in Fig.(6.2). Here the temperatures of stars are normalized to the sunny surface temperature (5875 C), the stars masses are normalized to the mass of the Sum. The data are shown on double logarithmic scale. The solid line shows the result of fitting of measurement data ($\mathbb{T}_0 \sim \mathbb{M}^{0.59}$). The theoretical dependence $\mathbb{T}_0 \sim \mathbb{M}^{7/12}$ (Eq.6.20) is shown by dotted line.

If parameters of the star are expressed through corresponding solar values $\tau \equiv \frac{\mathbb{T}_0}{\mathbb{T}_\odot}$ and $\mu \equiv \frac{\mathbb{M}}{\mathbb{M}_\odot}$, that Eq.(6.20) can be rewritten as

$$\frac{\tau}{\mu^{7/12}} = 1. \tag{6.21}$$

Numerical values of relations $\frac{\tau}{\mu^{7/12}}$ for close binary stars [11] are shown in the Table(6.2).

The analysis of these data leads to few conclusions. The averaging over all tabulated stars gives

$$< \frac{\tau}{\mu^{7/12}} >= 1.007 \pm 0.07. \tag{6.22}$$

and we can conclude that the variability of measured data of surface temperatures and stellar masses has statistical character. Secondly, Eq.(6.21) is valid for all hot stars (exactly for all stars which are gathered in Tab.(6.2)).

The problem with the averaging of $\frac{\rho}{\mu^{2/3}}$ looks different. There are a few of giants and super-giants in this Table. The values of ratio $\frac{\rho}{\mu^{2/3}}$ are more than 2 for them. It seems that, if to exclude these stars from consideration, the averaging over stars of the main sequence gives value close to 1. Evidently, it needs in more detail consideration.

The luminosity of a star

$$\mathbb{L}_0 \sim \mathbb{R}_0^2 \mathbb{T}_0^4. \tag{6.23}$$

at taking into account (Eq.6.16) and (Eq.6.20) can be expressed as

$$\mathbb{L}_0 \sim \mathbb{M}^{11/3} \sim \mathbb{M}^{3.67} \tag{6.24}$$

This dependence is shown in Fig.(6.3) It can be seen that all calculated interdependencies $\mathbb{R}(\mathbb{M})$,$\mathbb{T}(\mathbb{M})$ and L($\mathbb{M}$) show a good qualitative agreement with the measuring data. At that it is important, that the quantitative explanation of mass-luminosity dependence discovered at the beginning of 20th century is obtained.

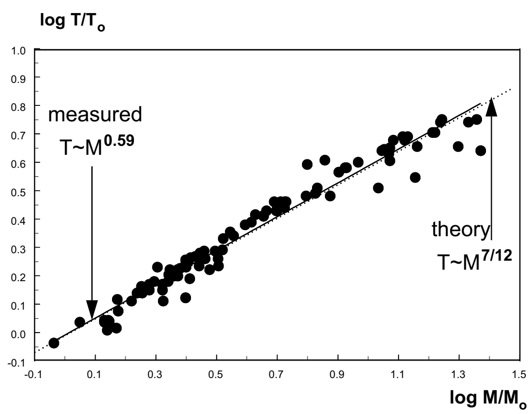

Figure 6.2: The dependence of the temperature on the star surface over the star mass of close binary stars [11]. Here the temperatures of stars are normalized to surface temperature of the Sun (5875 C), the stars masses are normalized to the mass of Sum. The data are shown on double logarithmic scale. The solid line shows the result of fitting of measurement data ($\mathbb{T}_0 \sim \mathbb{M}^{0.59}$). The theoretical dependence $\mathbb{T}_0 \sim \mathbb{M}^{7/12}$ (Eq.6.20) is shown by dotted line.

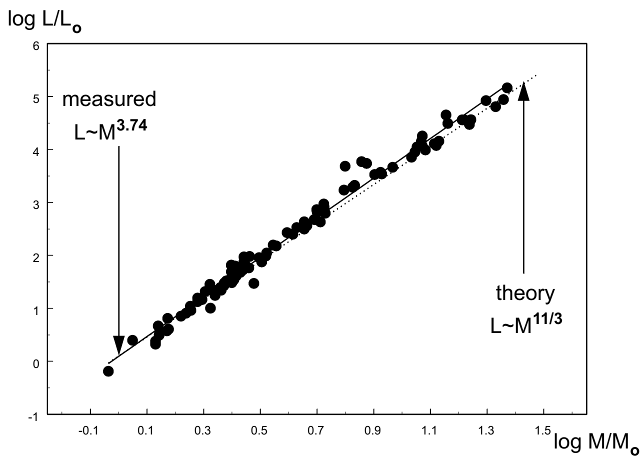

Figure 6.3: The dependence of star luminosity on the star mass of close binary stars [11]. The luminosities are normalized to the luminosity of the Sun, the stars masses are normalized to the mass of the Sum. The data are shown on double logarithmic scale. The solid line shows the result of fitting of measurement data $\mathbb{L} \sim \mathbb{M}^{3.74}$. The theoretical dependence $\mathbb{L} \sim \mathbb{M}^{11/3}$ (Eq.6.24) is shown by dotted line.

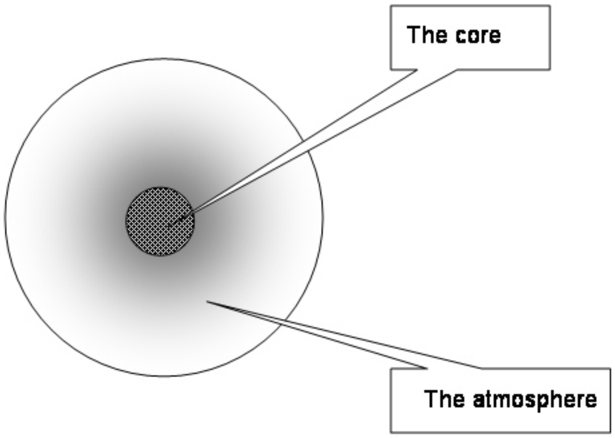

Figure 6.4: The schematic of the star interior

6.3.1 The compilation of the results of calculations

Let us put together the results of calculations. It is energetically favorable for the star to be divided into two volumes: the core is located in the central area of the star and the atmosphere is surrounding it from the outside. (Fig.6.4). The core has the radius:

$$\mathbb{R}_\star = 2.08 \frac{a_B}{Z(A/Z)} \left(\frac{\hbar c}{G m_p^2}\right)^{1/2} \approx \frac{1.41 \cdot 10^{11}}{Z(A/Z)} cm. \tag{6.25}$$

It is roughly equal to 1/10 of the stellar radius.

At that the mass of the core is equal to

$$\mathbb{M}_\star = 6.84 \frac{\mathbb{M}_{Ch}}{\left(\frac{A}{Z}\right)^2}. \tag{6.26}$$

It is almost exactly equal to one half of the full mass of the star.

The plasma inside the core has the constant density

$$n_\star = \frac{16}{9\pi}\frac{Z^3}{a_B^3} \approx 1.2 \cdot 10^{24} Z^3 cm^{-3} \tag{6.27}$$

and constant temperature

$$\mathbb{T}_\star = \left(\frac{25 \cdot 13}{28\pi^4}\right)^{1/3} \left(\frac{\hbar c}{k a_B}\right) Z \approx Z \cdot 2.13 \cdot 10^7 K. \tag{6.28}$$

The plasma density and its temperature are decreasing at an approaching to the stellar surface:

$$n_e(r) = n_\star \left(\frac{\mathbb{R}_\star}{r}\right)^6 \tag{6.29}$$

and

$$T_r = \mathbb{T}_\star \left(\frac{\mathbb{R}_\star}{r}\right)^4. \tag{6.30}$$

The external radius of the star is determined as

$$\mathbb{R}_0 = \left(\frac{\sqrt{\alpha\pi}}{2\eta}\frac{\frac{A}{Z}m_p}{m_e}\right)^{1/2} \mathbb{R}_\star \approx \frac{6.44 \cdot 10^{11}}{Z(A/Z)^{1/2}} cm \tag{6.31}$$

and the temperature on the stellar surface is equal to

$$\mathbb{T}_0 = \mathbb{T}_\star \left(\frac{\mathbb{R}_\star}{\mathbb{R}_0}\right)^4 \approx 4.92 \cdot 10^5 \frac{Z}{(A/Z)^2} \tag{6.32}$$

Chapter 7

Magnetic fields and magnetic moments of stars

7.1 Magnetic moments of celestial bodies

A thin spherical surface with radius r carrying an electric charge q at the rotation around its axis with frequency Ω obtains the magnetic moment

$$\mathfrak{m} = \frac{r^2}{3c} q\mathbf{\Omega}. \tag{7.1}$$

The rotation of a ball charged at density $\varrho(r)$ will induce the magnetic moment

$$\boldsymbol{\mu} = \frac{\mathbf{\Omega}}{3c} \int_0^R r^2 \varrho(r)\ 4\pi r^2 dr. \tag{7.2}$$

Thus the positively charged core of a star induces the magnetic moment

$$\mathfrak{m}_+ = \frac{\sqrt{G}\mathbb{M}_\star \mathbb{R}_\star^2}{5c}\mathbf{\Omega}. \tag{7.3}$$

A negative charge will be concentrated in the star atmosphere. The absolute value of atmospheric charge is equal to the positive charge of a core. As the atmospheric charge is placed near the surface of a star, its magnetic moment will be more than the core magnetic moment. The calculation shows that as a result, the total magnetic moment of the star will have the same order of magnitude as the core but it will be negative:

$$\mathfrak{m}_\Sigma \approx -\frac{\sqrt{G}}{c}\mathbb{M}_\star \mathbb{R}_\star^2 \mathbf{\Omega}. \tag{7.4}$$

Simultaneously, the torque of a ball with mass $\mathbb{M}$ and radius $\mathbb{R}$ is

$$\boldsymbol{\mathcal{L}} \approx \mathbb{M}_\star \mathbb{R}_\star^2 \boldsymbol{\Omega}. \tag{7.5}$$

As a result, for celestial bodies where the force of their gravity induces the electric polarization according to Eq.(4.2), the giromagnetic ratio will depend on world constants only:

$$\frac{\mathfrak{m}_\Sigma}{\boldsymbol{\mathcal{L}}} \approx -\frac{\sqrt{G}}{c}. \tag{7.6}$$

This relation was obtained for the first time by P.M.S.Blackett [6]. He shows that giromagnetic ratios of the Earth, the Sun and the star 78 Vir are really near to $\sqrt{G}/c$.

By now the magnetic fields, masses, radii and velocities of rotation are known for all planets of the Solar system and for a some stars [18]. These measuring data are shown in Fig.(7.1), which is taken from [18]. It is possible to see that these data are in satisfactory agreement with Blackett's ratio. At some assumption, the same parameters can be calculated for pulsars. All measured masses of pulsars are equal by the order of magnitude [21]. It is in satisfactory agreement with the condition of equilibrium of relativistic matter (see). It gives a possibility to consider that masses and radii of pulsars are determined. According to generally accepted point of view, pulsar radiation is related with its rotation, and it gives their rotation velocity. These assumptions permit to calculate the giromagnetic ratios for three pulsars with known magnetic fields on their poles [5]. It is possible to see from Fig.(7.1), the giromagnetic ratios of these pulsars are in agreement with Blackett's ratio.

7.2 Magnetic fields of hot stars

At the estimation of the magnetic field on the star pole, it is necessary to find the field which is induced by stellar atmosphere. The field which is induced by stellar core is small because $\mathbb{R}_\star \ll R_0$. The field of atmosphere

$$\mathfrak{m}_- = \frac{\boldsymbol{\Omega}}{3c}\int_{R_\star}^{R_0} 4\pi \frac{div\mathfrak{P}}{3} r^4 dr. \tag{7.7}$$

can be calculated numerically. But, for our purpose it is enough to estimate this field in order of value:

$$\boldsymbol{\mathcal{H}} \approx \frac{2\mathfrak{m}_-}{R_0^3}. \tag{7.8}$$

As

$$\mathfrak{m}_- \approx \frac{\sqrt{G} 2 M_\star R_0^2}{c}\boldsymbol{\Omega} \tag{7.9}$$

the field on the star pole

$$\boldsymbol{\mathcal{H}} \approx -4\frac{\sqrt{G}\mathbb{M}_\star}{cR_0}\boldsymbol{\Omega}. \tag{7.10}$$

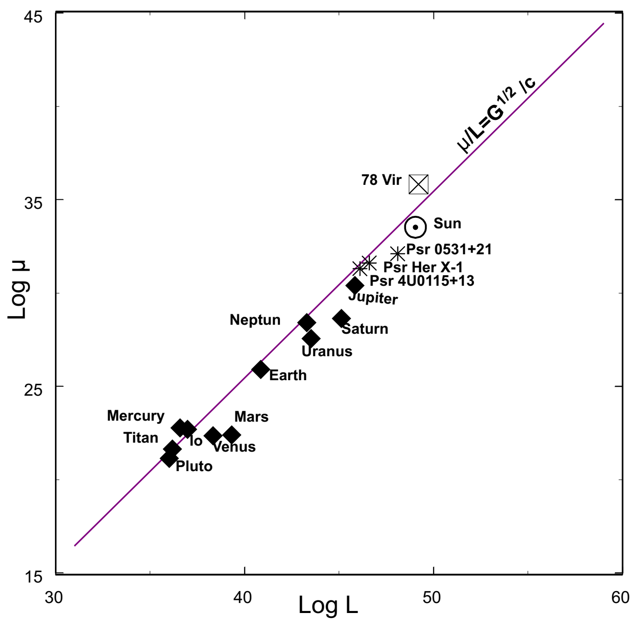

Figure 7.1: The observed values of magnetic moments of celestial bodies vs. their angular momenta [18]. In ordinate, the logarithm of the magnetic moment (in $Gs \cdot cm^3$) is plotted; in abscissa the logarithm of the angular momentum (in $erg \cdot s$) is shown. The solid line illustrates Eq.(13.2). The dash-dotted line fits of observed values.

At taking into account above obtained relations, one can see that this field is weakly depending on Z and A/Z, i.e. on the star temperature, on the star radius and mass. It depends linearly on the velocity of star rotation only:

$$\boldsymbol{\mathcal{H}} \approx -50\left(\frac{m_e}{m_p}\right)^{3/2}\frac{\alpha^{3/4}c}{\sqrt{G}}\boldsymbol{\Omega} \approx -2\cdot 10^9\boldsymbol{\Omega} \quad Oe. \tag{7.11}$$

The magnetic fields are measured for stars of Ap-class [16]. These stars are characterized by changing their brightness in time. The periods of these changes are measured too. At present there is no full understanding of causes of these visible changes of the luminosity. If these luminosity changes caused by some internal reasons will occur not uniformly on a star surface, one can conclude that the measured period of the luminosity change can depend on star rotation. It is possible to think that at relatively rapid rotation of a star, the period of a visible change of the luminosity can be determined by this rotation in general. To check this suggestion, we can compare the calculated dependence (Eq.7.11) with measuring data [16] (see Fig. 7.2). Evidently one must not expect very good coincidence of calculations and measuring data, because calculations were made for the case of a spherically symmetric model and measuring data are obtained for stars where this symmetry is obviously violated. So getting consent on order of the value can be considered as wholly satisfied. It should be said that Eq.(7.11) does not working well in case with the Sun. The Sun surface rotates with period $T \approx 25 \div 30$ days. At this velocity of rotation, the magnetic field on the Sun pole calculated accordingly to Eq.(7.11) must be about 1 kOe. The dipole field of Sun according to experts estimation is approximately 20 times lower. There can be several reasons for that.

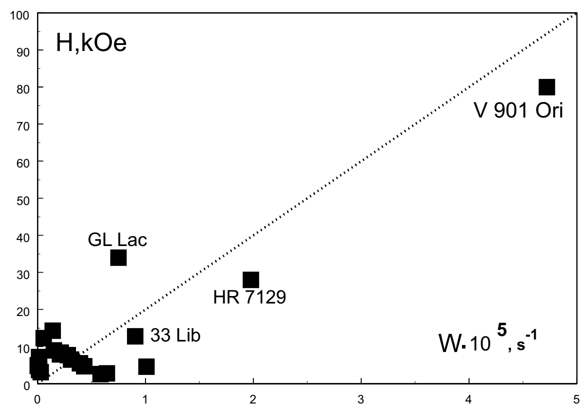

Figure 7.2: The dependence of magnetic fields on poles of Ap-stars as a function of their rotation velocity [16]. The line shows Eq.(7.11)).

Chapter 8

The angular velocity of the apsidal rotation in binary stars

8.1 The apsidal rotation of close binary stars

The apsidal rotation (or periastron rotation) of close binary stars is a result of their non-Keplerian movement which originates from the non-spherical form of stars. This non-sphericity has been produced by rotation of stars around their axes or by their mutual tidal effect. The second effect is usually smaller and can be neglected. The first and basic theory of this effect was developed by A.Clairault at the beginning of the XVIII century. Now this effect was measured for approximately 50 double stars. According to Clairault's theory the velocity of periastron rotation must be approximately 100 times faster if matter is uniformly distributed inside a star. Reversely, it would be absent if all star mass is concentrated in the star center. To reach an agreement between the measurement data and calculations, it is necessary to assume that the density of substance grows in direction to the center of a star and here it runs up to a value which is hundreds times greater than mean density of a star. Just the same mass concentration of the stellar substance is supposed by all standard theories of a star interior. It has been usually considered as a proof of astrophysical models. But it can be considered as a qualitative argument. To obtain a quantitative agreement between theory and measurements, it is necessary to fit parameters of the stellar substance distribution in each case separately.

Let us consider this problem with taking into account the gravity induced electric polarization of plasma in a star. As it was shown above, one half of full mass of a star is concentrated in its plasma core at a permanent density. Therefor, the effect of periastron rotation of close binary stars must be reviewed with the account of a change of forms of these star cores.

According to [7],[17] the ratio of the angular velocity ω of rotation of periastron which is produced by the rotation of a star around its axis with the angular velocity Ω is

$$\frac{\omega}{\Omega} = \frac{3}{2}\frac{(I_A - I_C)}{Ma^2} \tag{8.1}$$

where I_A and I_C are the moments of inertia relatively to principal axes of the ellipsoid. Their difference is

$$I_A - I_C = \frac{M}{5}(a^2 - c^2), \tag{8.2}$$

where a and c are the equatorial and polar radii of the star.

Thus we have

$$\frac{\omega}{\Omega} \approx \frac{3}{10}\frac{(a^2 - c^2)}{a^2}. \tag{8.3}$$

8.2 The equilibrium form of the core of a rotating star

In the absence of rotation the equilibrium equation of plasma inside star core (Eq.4.4 is

$$\gamma \mathbf{g}_G + \rho_G \mathbf{E}_G = 0 \tag{8.4}$$

where γ,$\mathbf{g}_G$, ρ_G and $\mathbf{E}_G$ are the substance density the acceleration of gravitation, gravity-induced density of charge and intensity of gravity-induced electric field ($div\ \mathbf{g}_G = 4\pi\ G\ \gamma$, $div\ \mathbf{E}_G = 4\pi\rho_G$ and $\rho_G = \sqrt{G}\gamma$).

One can suppose, that at rotation, under action of a rotational acceleration $\mathbf{g}_\Omega$, an additional electric charge with density ρ_Ω and electric field $\mathbf{E}_\Omega$ can exist, and the equilibrium equation obtains the form:

$$(\gamma_G + \gamma_\Omega)(\mathbf{g}_G + \mathbf{g}_\Omega) = (\rho_G + \rho_\Omega)(\mathbf{E}_G + \mathbf{E}_\Omega), \tag{8.5}$$

where

$$div\ (\mathbf{E}_G + \mathbf{E}_\Omega) = 4\pi(\rho_G + \rho_\Omega) \tag{8.6}$$

or

$$div\ \mathbf{E}_\Omega = 4\pi\rho_\Omega. \tag{8.7}$$

We can look for a solution for electric potential in the form

$$\varphi = C_\Omega\ r^2(3cos^2\theta - 1) \tag{8.8}$$

or in Cartesian coordinates

$$\varphi = C_\Omega(3z^2 - x^2 - y^2 - z^2) \tag{8.9}$$

where C_Ω is a constant.
Thus

$$E_x = 2\ C_\Omega\ x,\ E_y = 2\ C_\Omega\ y,\ E_z = -4\ C_\Omega\ z \tag{8.10}$$

and

$$div\ \mathbf{E}_\Omega = 0 \tag{8.11}$$

and we obtain important equations:

$$\rho_\Omega = 0; \tag{8.12}$$

$$\gamma g_\Omega = \rho \mathbf{E}_\Omega. \tag{8.13}$$

Since centrifugal force must be contra-balanced by electric force

$$\gamma\ 2\Omega^2\ x = \rho\ 2C_\Omega\ x \tag{8.14}$$

and

$$C_\Omega = \frac{\gamma\ \Omega^2}{\rho} = \frac{\Omega^2}{\sqrt{G}} \tag{8.15}$$

The potential of a positive uniform charged ball is

$$\varphi(r) = \frac{Q}{R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right) \tag{8.16}$$

The negative charge on the surface of a sphere induces inside the sphere the potential

$$\varphi(R) = -\frac{Q}{R} \tag{8.17}$$

where according to Eq.(8.4) $Q = \sqrt{G}M$, and M is the mass of the star.
Thus the total potential inside the considered star is

$$\varphi_\Sigma = \frac{\sqrt{G}M}{2R}\left(1 - \frac{r^2}{R^2}\right) + \frac{\Omega^2}{\sqrt{G}} r^2 (3cos^2\theta - 1) \tag{8.18}$$

Since the electric potential must be equal to zero on the surface of the star, at $r = a$ and $r = c$

$$\varphi_\Sigma = 0 \tag{8.19}$$

and we obtain the equation which describes the equilibrium form of the core of a rotating star (at $\frac{a^2-c^2}{a^2} \ll 1$)

$$\frac{a^2 - c^2}{a^2} \approx \frac{9}{2\pi}\frac{\Omega^2}{G\gamma}. \tag{8.20}$$

8.3 The angular velocity of the apsidal rotation

Taking into account of Eq.(8.20) we have

$$\frac{\omega}{\Omega} \approx \frac{27}{20\pi}\frac{\Omega^2}{G\gamma} \tag{8.21}$$

If both stars of a close pair induce a rotation of periastron, this equation transforms to

$$\frac{\omega}{\Omega} \approx \frac{27}{20\pi}\frac{\Omega^2}{G}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right), \tag{8.22}$$

where γ_1 and γ_2 are densities of star cores.

The equilibrium density of star cores is known (Eq.(2.18)):

$$\gamma = \frac{16}{9\pi^2}\frac{A}{Z}m_p\frac{Z^3}{a_B^3}. \tag{8.23}$$

If we introduce the period of ellipsoidal rotation $P = \frac{2\pi}{\Omega}$ and the period of the rotation of periastron $U = \frac{2\pi}{\omega}$, we obtain from Eq.(8.21)

$$\frac{\mathcal{P}}{\mathcal{U}}\left(\frac{\mathcal{P}}{\mathcal{T}}\right)^2 \approx \sum_1^2 \xi_i, \tag{8.24}$$

where

$$\mathcal{T} = \sqrt{\frac{243\ \pi^3}{80}}\ \tau_0 \approx 10\tau_0, \tag{8.25}$$

$$\tau_0 = \sqrt{\frac{a_B^3}{G\ m_p}} \approx 7.7\cdot 10^2 sec \tag{8.26}$$

and

$$\xi_i = \frac{Z_i}{A_i(Z_i+1)^3}. \tag{8.27}$$

8.4 The comparison of the calculated angular velocity of the periastron rotation with observations

Because the substance density (Eq.(8.23)) is depending approximately on the second power of the nuclear charge, the periastron movement of stars consisting of heavy elements will fall out from the observation as it is very slow. Practically the obtained equation (8.24) shows that it is possible to observe the periastron rotation of a star consisting of light elements only.

The value $\xi = Z/[AZ^3]$ is equal to 1/8 for hydrogen, 0.0625 for deuterium, $1.85\cdot 10^{-2}$ for helium. The resulting value of the periastron rotation of double stars will be the sum of separate stars rotation. The possible combinations of a couple and their value of $\sum_1^2 \xi_i$ for stars consisting of light elements is shown in Table 8.4.

star1 composed of	star2 composed of	$\xi_1 + \xi_2$
H	H	.25
H	D	0.1875
H	He	0.143
H	hn	0.125
D	D	0.125
D	He	0.0815
D	hn	0.0625
He	He	0.037
He	hn	0.0185

Table 8.4

The "hn" notation in Table 8.4 indicates that the second component of the couple consists of heavy elements or it is a dwarf.

The results of measuring of main parameters for close binary stars are gathered in [11]. For reader convenience, the data of these measurement is applied in the Table in Appendix. One can compare our calculations with data of these measurements. The distribution of close binary stars on value of $(\mathcal{P}/\mathcal{U})(\mathcal{P}/\mathcal{T})^2$ is shown in Fig.8.1 on logarithmic scale. The lines mark the values of parameters $\sum_1^2 \xi_i$ for different light atoms in accordance with 8.27. It can be seen that calculated values the periastron rotation for stars composed by light elements which is summarized in Table8.4 are in good agreement with separate peaks of measured data. It confirms that our approach to interpretation of this effect is adequate to produce a satisfactory accuracy of estimations.

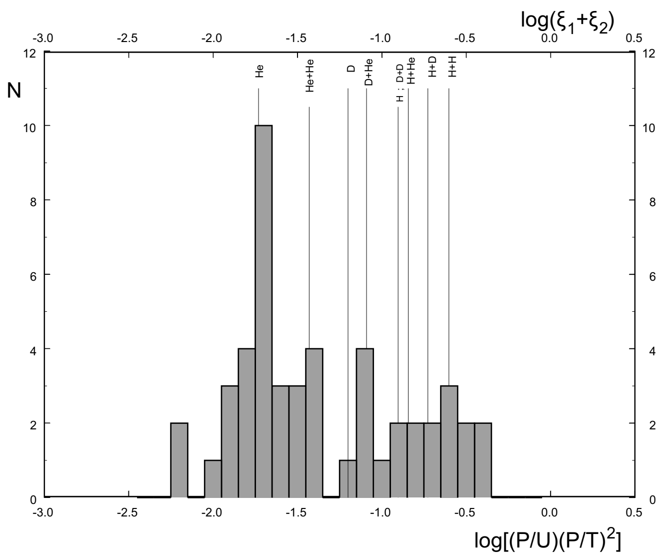

Figure 8.1: The distribution of close binary stars [11] on value of $(\mathcal{P}/\mathcal{U})(\mathcal{P}/\mathcal{T})^2$. Lines show parameters $\sum_1^2 \xi_i$ for different light atoms in according with 8.27.

Chapter 9

The solar seismical oscillations

9.1 The spectrum of solar seismic oscillations

The measurements [9] show that the Sun surface is subjected to a seismic vibration. The most intensive oscillations have the period about five minutes and the wave length about 10^4km or about hundredth part of the Sun radius. Their spectrum obtained by BISON collaboration is shown in Fig.9.1.

It is supposed, that these oscillations are a superposition of a big number of different modes of resonant acoustic vibrations, and that acoustic waves propagate in different trajectories in the interior of the Sun and they have multiple reflection from surface. With these reflections trajectories of same waves can be closed and as a result standing waves are forming.

Specific features of spherical body oscillations are described by the expansion in series on spherical functions. These oscillations can have a different number of wave lengths on the radius of a sphere (n) and a different number of wave lengths on its surface which is determined by the l-th spherical harmonic. It is accepted to describe the sunny surface oscillation spectrum as the expansion in series [8]:

$$\nu_{nlm} \simeq \Delta\nu_0(n + \frac{l}{2} + \epsilon_0) - l(l+1)D_0 + m\Delta\nu_{rot}. \tag{9.1}$$

Where the last item is describing the effect of the Sun rotation and is small. The main contribution is given by the first item which creates a large splitting in the spectrum (Fig.9.1)

$$\triangle\nu = \nu_{n+1,l} - \nu_{n,l}. \tag{9.2}$$

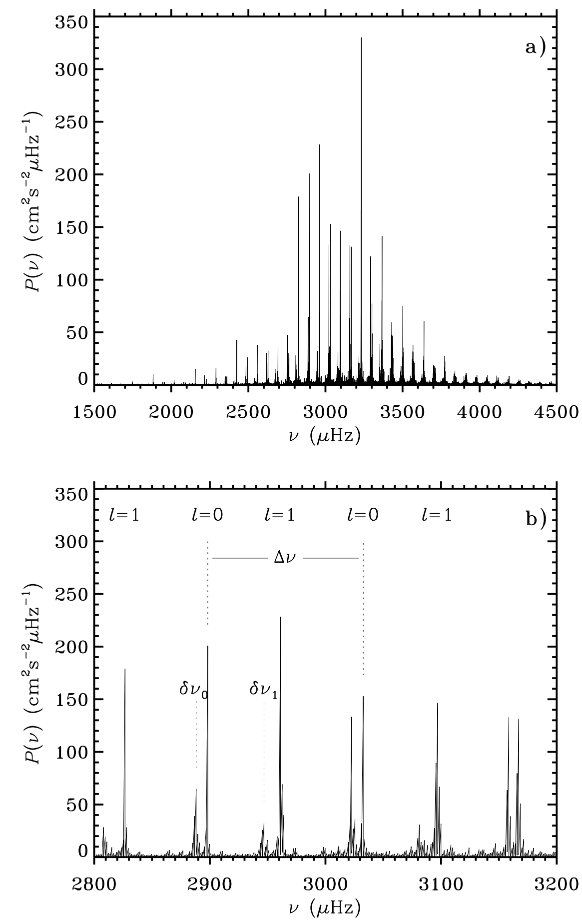

Figure 9.1: (a) The power spectrum of solar oscillation obtained by means of Doppler velocity measurement in light integrated over the solar disk. The data were obtained from the BISON network [9]. (b) An expanded view of a part of frequency range.

The small splitting of spectrum (Fig.9.1) depends on the difference

$$\delta\nu_l = \nu_{n,l} - \nu_{n-1,l+2} \approx (4l+6)D_0. \tag{9.3}$$

A satisfactory agreement of these estimations and measurement data can be obtained at [8]

$$\Delta\nu_0 = 120\ \mu Hz,\ \epsilon_0 = 1.2,\ D_0 = 1.5\ \mu Hz,\ \Delta\nu_{rot} = 1\mu Hz. \tag{9.4}$$

To obtain these values of parameters $\Delta\nu_0,\ \epsilon_0\ D_0$ from theoretical models is not possible. There are a lot of qualitative and quantitative assumptions used at a model construction and a direct calculation of spectral frequencies transforms into a unresolved complicated problem.

Thus, the current interpretation of the measuring spectrum by the spherical harmonic analysis does not make it clear. It gives no hint to an answer to the question: why oscillations close to hundredth harmonics are really excited and there are no waves near fundamental harmonic?

The measured spectra have a very high resolution (see Fig.(9.1)). It means that an oscillating system has high quality. At this condition, the system must have oscillation on a fundamental frequency. Some peculiar mechanism must exist to force a system to oscillate on a high harmonic. The current explanation does not clarify it.

It is important, that now the solar oscillations are measured by means of two different methods. The solar oscillation spectra which was obtained on program "BISON", is shown on Fig.(9.1)). It has a very high resolution, but (accordingly to the Liouville's theorem) it was obtained with some loss of luminosity, and as a result not all lines are well statistically worked.

Another spectrum was obtained in the program "SOHO/GOLF". Conversely, it is not characterized by high resolution, instead it gives information about general character of the solar oscillation spectrum (Fig.9.2)).

The existence of this spectrum requires to change the view at all problems of solar oscillations. The theoretical explanation of this spectrum must give answers at least to four questions :

1.Why does the whole spectrum consist from a large number of equidistant spectral lines?

2.Why does the central frequency of this spectrum $\mathcal{F}$ is approximately equal to $\approx 3.23\ mHz$?

3. Why does this spectrum splitting f is approximately equal to 67.5 μHz?

4. Why does the intensity of spectral lines decrease from the central line to the periphery?

The answers to these questions can be obtained if we take into account electric polarization of a solar core.

The description of measured spectra by means of spherical analysis does not make clear of the physical meaning of this procedure. The reason of difficulties lies in attempt to consider the oscillations of a Sun as a whole. At existing dividing of a star into core and atmosphere, it is easy to understand that the core oscillation must form a

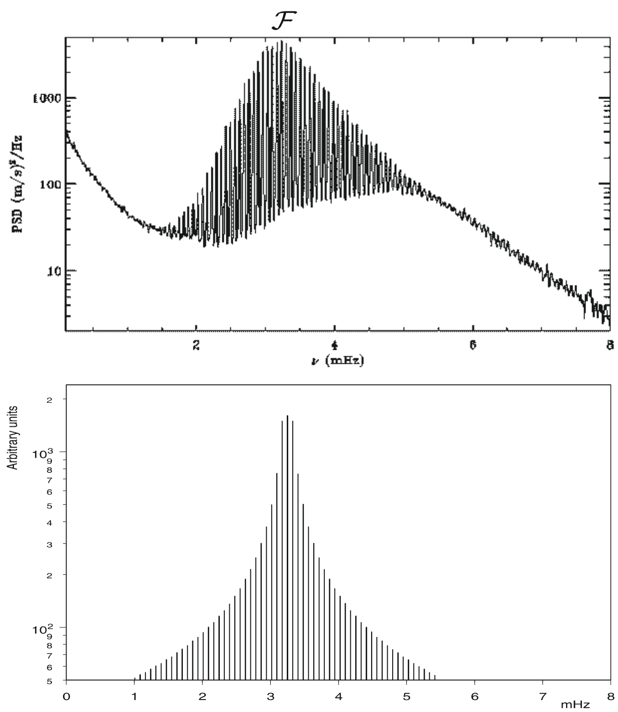

Figure 9.2: (a) The measured power spectrum of solar oscillation. The data were obtained from the SOHO/GOLF measurement [19]. (b) The calculated spectrum described by Eq.(9.26) at $< Z >= 3.4$ and $A/Z = 5$.

measured spectrum. The fundamental mode of this oscillation must be determined by its spherical mode when the Sun radius oscillates without changing of the spherical form of the core. It gives a most low-lying mode with frequency:

$$\Omega_s \approx \frac{c_s}{\mathbb{R}_\star}, \tag{9.5}$$

where c_s is sound velocity in the core.

It is not difficult to obtain the numerical estimation of this frequency by order of magnitude. Supposing that the sound velocity in dense matter is $10^7 cm/c$ and radius is close to $\frac{1}{10}$ of external radius of a star, i.e. about $10^{10} cm$, one can obtain as a result

$$F = \frac{\Omega_s}{2\pi} \approx 10^{-3} Hz \tag{9.6}$$

It gives possibility to conclude that this estimation is in agreement with measured frequencies. Let us consider this mechanism in more detail.

9.2 The sound speed in hot plasma

The pressure of high temperature plasma is a sum of the plasma pressure (ideal gas pressure) and the pressure of black radiation:

$$P = n_e kT + \frac{\pi^2}{45\hbar^3 c^3}(kT)^4. \tag{9.7}$$

and its entropy is

$$S = \frac{1}{\frac{A}{Z} m_p} \, ln \, \frac{(kT)^{3/2}}{n_e} + \frac{4\pi^2}{45\hbar^3 c^3 n_e}(kT)^3, \tag{9.8}$$

The sound speed c_s can be expressed by Jacobian [12]:

$$c_s^2 = \frac{D(P,S)}{D(\rho,S)} = \frac{\left(\frac{D(P,S)}{D(n_e,T)}\right)}{\left(\frac{D(\rho,S)}{D(n_e,T)}\right)} \tag{9.9}$$

or

$$c_s = \left\{ \frac{5}{9} \frac{kT}{A/Z m_p} \left[1 + \frac{2\left(\frac{4\pi^2}{45\hbar^3 c^3}\right)^2 (kT)^6}{5 n_e [n_e + \frac{8\pi^2}{45\hbar^3 c^3}(kT)^3]} \right] \right\}^{1/2} \tag{9.10}$$

For $T = \mathbb{T}_\star$ and $n_e = n_\star$ we have:

$$\frac{4\pi^2 (k\mathbb{T}_\star)^3}{45\hbar^3 c^3 n_\star} = \approx 0.18 \ . \tag{9.11}$$

Finally we obtain:

$$c_s = \left\{ \frac{5}{9} \frac{\mathbb{T}_\star}{(A/Z) m_p} [1.01] \right)^{1/2} \approx 3.14 \ 10^7 \left(\frac{Z}{A/Z} \right)^{1/2} cm/s \ . \tag{9.12}$$

9.3 The basic elastic oscillation of a spherical core

Star cores consist of dense high temperature plasma which is a compressible matter. The basic mode of elastic vibrations of a spherical core is related with its radius oscillation. For the description of this type of oscillation, the potential ϕ of displacement velocities $v_r = \frac{\partial \psi}{\partial r}$ can be introduced and the motion equation can be reduced to the wave equation expressed through ϕ [12]:

$$c_s^2 \Delta \phi = \ddot{\phi}, \tag{9.13}$$

and a spherical derivative for periodical in time oscillations ($\sim e^{-i\Omega_s t}$) is:

$$\Delta \phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) = -\frac{\Omega_s^2}{c_s^2}\phi \ . \tag{9.14}$$

It has the finite solution for the full core volume including its center

$$\phi = \frac{A}{r} sin \frac{\Omega_s r}{c_s}, \tag{9.15}$$

where A is a constant. For small oscillations, when displacements on the surface u_R are small ($u_R/R = v_R/\Omega_s R \to 0$) we obtain the equation:

$$tg \frac{\Omega_s \mathbb{R}}{c_s} = \frac{\Omega_s \mathbb{R}}{c_s} \tag{9.16}$$

which has the solution:

$$\frac{\Omega_s \mathbb{R}}{c_s} \approx 4.49. \tag{9.17}$$

Taking into account Eq.(9.12)), the main frequency of the core radial elastic oscillation is

$$\Omega_s = 4.49 \left\{ 1.4 \left[\frac{G m_p}{r_B^3}\right] \frac{A}{Z} \left(Z+1\right)^3 \right\}^{1/2}. \tag{9.18}$$

It can be seen that this frequency depends on Z and A/Z only. Some values of frequencies of radial sound oscillations $\mathcal{F} = \Omega_s/2\pi$ calculated from this equation for selected A/Z and Z are shown in third column of Table (9.3).

Table (9.3)

Z	A/Z	$\mathcal{F}, mHz$ (calculated (9.18))	star	$\mathcal{F}, mHz$ measured
1	1	0.23	ξ *Hydrae*	~ 0.1
1	2	0.32	ν *Indus*	0.3
2	2	0.9	η *Bootis*	0.85
2	3	1.12	The Procion($A\alpha$ *CMi*)	1.04
			β *Hydrae*	1.08
3	4	2.38	α *Cen A*	2.37
3	5	2.66		
3.4	5	**3.24**	The Sun	**3.23**
4	5	4.1		

The star mass spectrum (.5.1) shows that the ratio A/Z must be $\approx$ 5 for the Sum. It is in accordance with the calculated frequency of solar core oscillations if the averaged charge of nuclei $Z \approx 3.4$. It is not a confusing conclusion, because the plasma electron gas prevents the decay of β-active nuclei (see Sec.10). This mechanism can probably to stabilize neutron-excess nuclei.

9.4 The low frequency oscillation of the density of a neutral plasma

Hot plasma has the density $n_\star$ at its equilibrium state. The local deviations from this state induce processes of density oscillation since plasma tends to return to its steady-state density. If we consider small periodic oscillations of core radius

$$R = \mathbb{R} + u_R \cdot sin\ \omega_{n_\star} t, \tag{9.19}$$

where a radial displacement of plasma particles is small ($u_R \ll \mathbb{R}$), the oscillation process of plasma density can be described by the equation

$$\frac{d\mathcal{E}}{dR} = \mathbb{M}\ddot{R}\ . \tag{9.20}$$

Taking into account

$$\frac{d\mathcal{E}}{dR} = \frac{d\mathcal{E}_{plasma}}{dn_e}\frac{dn_e}{dR} \tag{9.21}$$

and

$$\frac{3}{8}\pi^{3/2}\mathbb{N}_e \frac{e^3 a_0^{3/2}}{(k\mathbb{T})^{1/2}}\frac{n_\star}{\mathbb{R}^2} = \mathbb{M}\omega_{n_\star}^2 \tag{9.22}$$

From this we obtain

$$\omega_{n_\star}^2 = \frac{3}{\pi^{1/2}}k\mathbb{T}\left(\frac{e^2}{a_B k\mathbb{T}}\right)^{3/2}\frac{Z^3}{\mathbb{R}^2 A/Z m_p} \tag{9.23}$$

and finally

$$\omega_{n_\star} = \left\{\frac{2^8}{3^5}\frac{\pi^{1/2}}{10^{1/2}}\alpha^{3/2}\left[\frac{Gm_p}{a_B^3}\right]\frac{A}{Z}Z^{4.5}\right\}^{1/2}, \tag{9.24}$$

where $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant. These low frequency oscillations of neutral plasma density are similar to phonons in solid bodies. At that oscillations with multiple frequencies $k\omega_{n_\star}$ can exist. Their power is proportional to $1/\kappa$, as the occupancy these levels in energy spectrum must be reversely proportional to their energy $k\hbar\omega_{n_\star}$. As result, low frequency oscillations of plasma density constitute set of vibrations

$$\sum_{\kappa=1}\frac{1}{\kappa}\ sin(\kappa\omega_{n_\star} t)\ . \tag{9.25}$$

9.5 The spectrum of solar core oscillations

The set of the low frequency oscillations with ω_η can be induced by sound oscillations with Ω_s. At that, displacements obtain the spectrum:

$$u_R \sim \sin\ \Omega_s t \cdot \sum_{\kappa=0} \frac{1}{\kappa}\ \sin\ \kappa\omega_{n_\star} t \cdot \sim \xi \sin\ \Omega_s t + \sum_{\kappa=1} \frac{1}{\kappa}\ \sin\ (\Omega_s \pm \kappa\omega_{n_\star})t, \qquad (9.26)$$

where ξ is a coefficient ≈ 1.

This spectrum is shown in Fig.(9.2).

The central frequency of experimentally measured distribution of solar oscillations is approximately equal to (Fig.(9.1))

$$\mathcal{F}_\odot \approx 3.23\ mHz \qquad (9.27)$$

and the experimentally measured frequency splitting in this spectrum is approximately equal to

$$f_\odot \approx 68\ \mu Hz. \qquad (9.28)$$

A good agreement of the calculated frequencies of basic modes of oscillations (from Eq.(9.18) and Eq.(8.4)) with measurement can be obtained at $Z = 3.4$ and $A/Z = 5$:

$$\mathcal{F}_{Z=3.4;\frac{A}{Z}=5} = \frac{\Omega_s}{2\pi} = 3.24\ mHz;\ f_{Z=3.4;\frac{A}{Z}=5} = \frac{\omega_{n_\star}}{2\pi} = 68.1\ \mu Hz. \qquad (9.29)$$

Chapter 10

A mechanism of stabilization for neutron-excess nuclei in plasma

10.1 Neutron-excess nuclei and the neutronization

The form of the star mass spectrum (Fig.5.1) indicates that plasma of many stars consists of neutron-excess nuclei with the ratio $A/Z = 3,4,5$ and so on. These nuclei are subjects of a decay under the "terrestrial" conditions. Hydrogen isotopes ${}^4_1H$, ${}^5_1H$, ${}^6_1H$ have short time of life and emit electrons with energy more than 20 Mev. The decay of helium isotopes ${}^6_2He$, ${}^8_2He$, ${}^{10}_2He$ have the times of life, which can reach tenth part of the seconds.

Stars have the time of life about billions years and the lines of their spectrum of masses are not smoothed. Thus we should suppose that there is some mechanism of stabilization of neutron-excess nuclei inside stars. This mechanism is well known - it is neutronization [12]§106. It is accepted to think that this mechanism is characteristic for dwarfs with density of particles about 10^{30} per cm^3 and pressure of relativistic electron gas

$$P \approx \hbar c \cdot n_e^{4/3} \approx 10^{23} dyne/cm^2. \tag{10.1}$$

The possibility of realization of neutronization in dense plasma is considered below in detail. At thus, we must try to find an explanation to characteristic features of the star mass spectrum. At first, we can see that, there is actually quite a small number of stars with $A/Z = 2$ exactly. The question is arising: why there are so few stars, which are composed by very stable nuclei of helium-4? At the same time, there are many stars with $A/Z = 4$, i.e. consisting apparently of a hydrogen-4, as well as stars with $A/Z = 3/2$, which hypothetically could be composed by another isotope of helium - helium-3.

10.1.1 The electron cloud of plasma cell

Let us consider a possible mechanism of the action of the electron gas effect on the plasma nuclear subsystem. It is accepted to consider dense plasma to be divided in plasma cells. These cells are filled by electron gas and they have positively charged nuclei in their centers [15].

This construction is non stable from the point of view of the classical mechanics because the opposite charges collapse is "thermodynamic favorable". To avoid a divergence in the theoretical description of this problem, one can artificially cut off the integrating at some small distance characterizing the particles interaction. For example, nuclei can be considered as hard cores with the finite radii.

It is more correctly, to consider this structure as a quantum-mechanical object and to suppose that the electron can not approach the nucleus closer than its own de Broglie's radius λ_e.

Let us consider the behavior of the electron gas inside the plasma cell. If to express the number of electrons in the volume V through the density of electron n_e, then the maximum value of electron momentum [12]:

$$p_F = \left(3\pi^2\ n_e\right)^{1/3}\hbar. \tag{10.2}$$

The kinetic energy of the electron gas can be founded from the general expression for the energy of the Fermi-particles, which fills the volume V [12]:

$$\mathcal{E} = \frac{Vc}{\pi^2\hbar^3}\int_0^{p_F} p^2\sqrt{m_e^2c^2+p^2}dp. \tag{10.3}$$

After the integrating of this expression and the subtracting of the energy at rest, we can calculate the kinetic energy of the electron:

$$\mathcal{E}_{kin} = \frac{3}{8}m_e c^2\left[\frac{\xi(2\xi^2+1)\sqrt{\xi^2+1} - Arcsinh(\xi) - \frac{8}{3}\xi^3}{\xi^3}\right] \tag{10.4}$$

(where $\xi = \frac{p_F}{m_e c}$).

The potential energy of an electron is determined by the value of the attached electric field. The electrostatic potential of this field $\varphi(r)$ must be equal to zero at infinity.[1] With this in mind, we can write the energy balance equation of electron

$$\mathcal{E}_{kin} = e\varphi(r). \tag{10.5}$$

The potential energy of an electron at its moving in an electric field of the nucleus can be evaluated on the basis of the Lorentz transformation [14]§24. If in the laboratory frame of reference, where an electric charge placed, it creates an electric potential φ_0, the potential in the frame of reference moving with velocity v is

$$\varphi = \frac{\varphi_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{10.6}$$

Therefore, the potential energy of the electron in the field of the nucleus can be written as:

$$\mathcal{E}_{pot} = -\frac{Ze^2}{r}\frac{\xi}{\beta}. \tag{10.7}$$

Where

$$\beta = \frac{v}{c}. \tag{10.8}$$

and

$$\xi \equiv \frac{p}{m_e c}, \tag{10.9}$$

m_e is the mass of electron in the rest.

And one can rewrite the energy balance Eq.(10.5) as follows:

$$\frac{3}{8}m_e c^2 \xi \mathbb{Y} = e\varphi(r)\frac{\xi}{\beta}. \tag{10.10}$$

where

$$\mathbb{Y} = \left[\frac{\xi(2\xi^2+1)\sqrt{\xi^2+1} - Arcsinh(\xi) - \frac{8}{3}\xi^3}{\xi^4}\right]. \tag{10.11}$$

Hence

$$\varphi(r) = \frac{3}{8}\frac{m_e c^2}{e}\beta\mathbb{Y}. \tag{10.12}$$

In according with Poisson's electrostatic equation

$$\Delta\varphi(r) = 4\pi e n_e \tag{10.13}$$

or at taking into account that the electron density is depending on momentum (Eq.(10.2)), we obtain

$$\Delta\varphi(r) = \frac{4e}{3\pi}\left(\frac{\xi}{\lambda\!\!\!^{-}_C}\right)^3, \tag{10.14}$$

[1] In general, if there is an uncompensated electric charge inside the cell, then we would have to include it to the potential $\varphi(r)$. However, we can do not it, because will consider only electro-neutral cell, in which the charge of the nucleus exactly offset by the electronic charge, so the electric field on the cell border is equal to zero.

where $ƛ_C = \frac{\hbar}{m_e c}$ is the Compton radius.

At introducing of the new variable

$$\varphi(r) = \frac{\chi(r)}{r}, \tag{10.15}$$

we can transform the Laplacian:

$$\Delta\varphi(r) = \frac{1}{r}\frac{d^2\chi(r)}{dr^2}. \tag{10.16}$$

As (Eq.(10.12))

$$\chi(r) = \frac{3}{8}\frac{m_e c^2}{e}\mathbb{Y}\beta r \ , \tag{10.17}$$

the differential equation can be rewritten:

$$\frac{d^2\chi(r)}{dr^2} = \frac{\chi(r)}{\mathbb{L}^2} \ , \tag{10.18}$$

where

$$\mathbb{L} = \left(\frac{9\pi}{32}\frac{\mathbb{Y}\beta}{\alpha\xi^3}\right)^{1/2} ƛ_C, \tag{10.19}$$

$\alpha = \frac{1}{137}$ is the fine structure constant.

With taking in to account the boundary condition, this differential equation has the solution:

$$\chi(r) = C \cdot exp\left(-\frac{r}{\mathbb{L}}\right). \tag{10.20}$$

Thus, the equation of equilibrium of the electron gas inside a cell (Eq.(10.10)) obtains the form:

$$\frac{Ze}{r} \cdot e^{-r/\mathbb{L}} = \frac{3}{8} m_e c^2 \beta \mathbb{Y} \ . \tag{10.21}$$

10.2 The Thomas-Fermi screening

Let us consider the case when an ion is placed at the center of a cell, the external shells don't permit the plasma electron to approach to the nucleus on the distances much smaller than the Bohr radius. The electron moving is non-relativistic in this case. At that $\xi \to 0$, the kinetic energy of the electron

$$\mathcal{E}_{kin} = \frac{3}{8} m_e c^2 \xi \mathbb{Y} \to \frac{3}{5} E_F \ , \tag{10.22}$$

and the screening length

$$\mathbb{L} \to \sqrt{\frac{\mathcal{E}_F}{6\pi e^2 n_e}}. \tag{10.23}$$

Thus, we get the Thomas-Fermi screening in the case of the non-relativistic motion of an electron.

10.3 The screening with relativistic electrons

In the case the ≪bare≫ nucleus, there is nothing to prevent the electron to approach it at an extremely small distance λ_{min}, which is limited by its own than its de Broglie's wavelength. Its movement in this case becomes relativistic at $\beta \to 1 \;\; \xi \gg 1$. In this case, at not too small ξ, we obtain

$$\mathbb{Y} \approx 2\left(1 - \frac{4}{3\xi}\right), \qquad (10.24)$$

and at $\xi \gg 1$

$$\mathbb{Y} \to 2\ . \qquad (10.25)$$

In connection with it, at the distance $r \to \lambda_{min}$ from a nucleus, the equilibrium equation (10.21) reforms to

$$\lambda_{min} \simeq Z\alpha\lambda_C \quad . \qquad (10.26)$$

and the density of electron gas in a layer of thickness λ_{min} can be determined from the condition of normalization. As there are Z electrons into each cell, so

$$Z \simeq n_e^{\lambda} \cdot {\lambda_{min}}^3 \qquad (10.27)$$

From this condition it follows that

$$\xi_\lambda \simeq \frac{1}{2\alpha Z^{2/3}} \qquad (10.28)$$

Where n_e^{λ} and ξ_λ are the density of electron gas and the relative momentum of electrons at the distance λ_{min} from the nucleus. In accordance with Eq.(10.4), the energy of all the Z electrons in the plasma cell is

$$\mathcal{E} \simeq Z m_e c^2 \xi_\lambda \qquad (10.29)$$

At substituting of Eq.(10.28), finally we obtain the energy of the electron gas in a plasma cell:

$$\mathcal{E} \simeq \frac{m_e c^2}{2\alpha} Z^{1/3} \qquad (10.30)$$

This layer provides the pressure on the nucleus:

$$P \simeq \mathcal{E}\left(\frac{\xi}{\bar{\lambda}_C}\right)^3 \approx 10^{23} dyne/cm^2 \qquad (10.31)$$

This pressure is in order of value with pressure of neutronization (10.1).

Thus the electron cloud forms a barrier at a distance of $\alpha\left(\frac{\hbar}{m_e c}\right)$ from the nucleus. This barrier is characterized by the energy:

$$\mathcal{E} \simeq cp^{max} \simeq \frac{m_e c^2}{\alpha} \approx 70 Mev. \qquad (10.32)$$

This energy is many orders of magnitude more energy characteristic of the electron cloud on the periphery of the plasma cells, which we have neglected for good reason.

The barrier of the electron cloud near the nucleus will prevent its β-decay, if it has less energy. As a result, the nucleus that exhibit β-activity in the atomic matter will not disintegrate in plasma.

Of course, this barrier can be overcome due to the tunnel effect. The nuclei with large energy of emitted electrons have more possibility for decay. Probably it can explain the fact that the number of stars, starting with the $A/Z \approx 4$ spectrum Fig.(5.1), decreases continuously with increasing A/Z and drops to zero when $A/Z \approx 10$, where probably the decaying electron energy is approaching to the threshold Eq.(10.32).

It is interesting whether is possible to observe this effect in the laboratory? It seems that one could try to detect a difference in the rate of decay of tritium nuclei adsorbed in a metal. Hydrogen adsorbed in various states with different metals [20]. Hydrogen behaves like a halogen in the alkali metals (Li, K). It forms molecules H^-Li^+ and H^-K^+ with an absorbtion of electron from electron gas. Therefore, the tritium decay rate in such metals should be the same as in a molecular gas. However assumed [20], that adsorbed hydrogen is ionized in some metals, such as Ti, and it exists there in the form of gas of "naked" nuclei. In this case, the free electrons of the metal matrix should form clouds around the bare nuclei of tritium. In accordance with the above calculations, they should suppress the β-decay of tritium.

10.4 The neutronization

. The considered above ≪ attachment≫ of the electron to the nucleus in a dense plasma should lead to a phenomenon neutronization of the nucleus, if it is energetically favorable. The ≪ attached ≫ electron layer should have a stabilizing effect on the neutron-excess nuclei, i.e. the neutron-excess nucleus, which is instable into substance with the atomic structure, will became stable inside the dense plasma. It explains the stable existence of stars with a large ratio of A/Z.

These formulas allow to answer questions related to the characteristics of the star mass distribution. The numerical evaluation of energy the electron gas in a plasma cell gives:

$$\mathcal{E} \simeq \frac{m_e c^2}{2\alpha} Z^{1/3} \approx 5 \cdot 10^{-5} Z^{1/3} erg \tag{10.33}$$

The mass of nucleus of helium-4 $M(^4_2He) = 4.0026$ $a.e.m.$, and the mass hydrogen-4 $M(^4_1H) = 4.0278$ $a.e.m.$. The mass defect $\approx 3.8 \cdot 10^{-5} egr$. Therefore, the reaction

$$^4_2He + e \rightarrow ^4_1 H + \widetilde{\nu}, \tag{10.34}$$

is energetically favorable. At this reaction the nucleus captures the electron from gas and proton becomes a neutron.

There is the visible line of stars with the ratio $A/Z = 3/2$ at the star mass spectrum. It can be attributed to the stars, consisting of ^3_2He, ^6_4Be, ^9_6C, etc.

It is not difficult to verify at direct calculation that the reactions of neutronization and transforming of ^3_2He into ^3_1H and ^6_4Be into ^6_3Li are energetically allowed. So the nuclei ^3_2He and ^6_4Be should be conversed by neutronization into ^3_1H and ^6_3Li. The line $A/Z = 3/2$ of the star mass spectrum can not be formed by these nuclei. However, the reaction

$$^9_6C + e \rightarrow ^9_5 B \ + \widetilde{\nu}, \tag{10.35}$$

is not energetically allowed and therefore it is possible to believe that the stars of the above line mass spectrum may consist of carbon-9.

The mechanism of neutronization acting in the non-degenerate dense plasma and described above in this chapter seems quite realistic. However, the last nuclear reaction of neutronization can only be considered as hypothetical possibility and requires further more careful study.

Chapter 11

Other stars, their classification and some cosmology

The Schwarzsprung-Rassel diagram is now a generally accepted base for star classification. It seems that a classification based on the EOS of substance may be more justified from physical point of view. It can be emphasized by possibility to determine the number of classes of celestial bodies.

The matter can totally have eight states (Fig.(11.1)).

The atomic substance at low temperature exists as condensed matter (solid or liquid). At high temperature it transforms into gas phase.

The electron -nuclear plasma can exist in four states. It can be relativistic or non-relativistic. The electron gas of non-relativistic plasma can be degenerate (cold) or non-degenerate (hot). The relativistic electron gas is degenerate at temperature below T_F. Very high temperature can remove the degeneration even for relativistic electrons (if, certainly, electronic gas was not ultra-relativistic originally).

In addition to that, a substance can exist as neutron matter with the nuclear density approximately.

At present, assumptions about existence of matter at different states, other than the above-named, seem unfounded. Thus, these above-named states of matter show a possibility of classification of celestial bodies in accordance with this dividing.

	Low T	High T
Atomic substance	Solid body	gas
Plasma $p_F \ll m_e c$	Non-relativistic degenerate	Non-relarivistic Non-degenerate
Plasma $p_F \approx m_e c$	relativistic degenerate	relativistic non-degenerate
neutron matter $p_F \approx m_n c$	relativistic degenerate	relativistic non-degenerate

Figure 11.1: The steady states of substance

11.1 The atomic substance

11.1.1 Small bodies

Small celestial bodies - asteroids and satellites of planets - are usually considered as bodies consisting from atomic matter.

11.1.2 Giants

The transformation of atomic matter into plasma can be induced by action of high pressure, high temperature or both these factors. If these factors inside a body are not high enough, atomic substance can transform into gas state. The characteristic property of this celestial body is absence of electric polarization inside it. If temperature of a body is below ionization temperature of atomic substance but higher than its evaporation temperature, the equilibrium equation comes to

$$-\frac{dP}{dr} = \frac{G\gamma}{r^2} M_r \approx \frac{P}{R} \approx \frac{\gamma}{m_p} \frac{kT}{R}. \tag{11.1}$$

Thus, the radius of the body

$$R \approx \frac{GMm_p}{kT}. \tag{11.2}$$

If its mass $M \approx 10^{33}$ g and temperature at its center $T \approx 10^5$ K, its radius is $R \approx 10^2$ $R_\odot$. These proporties are characteristic for giants, where pressure at center is about $P \approx 10^{10}$ din/cm^2 and it is not enough for substance ionization.

11.2 Plasmas

11.2.1 The non-relativistic non-degenerate plasma. Stars.

Characteristic properties of hot stars consisting of non-relativistic non-degenerate plasma was considered above in detail. Its EOS is ideal gas equation.

11.2.2 Non-relativistic degenerate plasma. Planets.

At cores of large planets, pressures are large enough to transform their substance into plasma. As temperatures are not very high here, it can be supposed, that this plasma can degenerate:

$$T << T_F \tag{11.3}$$

The pressure which is induced by gravitation must be balanced by pressure of non-relativistic degenerate electron gas

$$\frac{G\mathbb{M}^2}{6\mathbb{R}\mathbb{V}} \approx \frac{(3\pi^2)^{2/3}}{5} \frac{\hbar^2}{m_e} \left(\frac{\gamma}{m_p A/Z}\right)^{5/3} \tag{11.4}$$

It opens a way to estimate the mass of this body:

$$\mathbb{M} \approx \mathbb{M}_{Ch}\left(\frac{\hbar}{mc}\right)^{3/2}\left(\frac{\gamma}{m_p}\right)^{1/2}\frac{6^{3/2}9\pi}{4(A/Z)^{5/2}} \tag{11.5}$$

At density about $\gamma \approx 1\ g/cm^3$, which is characteristic for large planets, we obtain their masses

$$\mathbb{M} \approx 10^{-3}\frac{\mathbb{M}_{Ch}}{(A/Z)^{5/2}} \approx \frac{4\cdot 10^{30}}{(A/Z)^{5/2}}\ g \tag{11.6}$$

Thus, if we suppose that large planets consist of hydrogen (A/Z=1), their masses must not be over $4\cdot 10^{30}g$. It is in agreement with the Jupiter's mass, the biggest planet of the Sun system.

11.2.3 The cold relativistic substance

The ratio between the main parameters of hot stars was calculated with the using of the virial theorem in the chapter [5]. It allowed us to obtain the potential energy of a star, consisting of a non-relativistic non-degenerate plasma at equilibrium conditions. This energy was obtained with taking into account the gravitational and electrical energy contribution. Because this result does not depend on temperature and plasma density, we can assume that the obtained expression of the potential energy of a star is applicable to the description of stars, consisting of a degenerate plasma. At least obtained expression Eq.(5.13) should be correct in this case by order of magnitude. Thus, in view of Eq.(4.20) and Eq.(5.36), the potential energy of the cold stars on the order of magnitude:

$$\mathcal{E}^{potential} \approx -\frac{G\mathbb{M}_{Ch}^2}{\mathbb{R}_0}. \tag{11.7}$$

A relativistic degenerate electron-nuclear plasma. Dwarfs

It is characteristic for a degenerate relativistic plasma that the electron subsystem is relativistic, while the nuclear subsystem can be quite a non-relativistic, and the main contribution to the kinetic energy os star gives its relativistic electron gas. Its energy was obtained above (Eq.(10.4)).

The application of the virial theorem gives the equation describing the existence of equilibrium in a star consisting of cold relativistic plasma:

$$\left[\xi(2\xi^2+1)\sqrt{\xi^2+1} - Arcsinh(\xi) - \frac{8}{3}\xi^3\right] \approx \xi, \tag{11.8}$$

This equation has a solution

$$\xi \approx 1. \tag{11.9}$$

The star, consisting of a relativistic non-degenerate plasma, in accordance with Eq.((10.2)), must have an electronic density

$$n_e \approx 5\cdot 10^{29} cm^{-3} \tag{11.10}$$

while the radius of the star will

$$R \approx 10^{-2} R_{\ odot}. \tag{11.11}$$

Easy to see that the density of matter and the radius are characteristic of cosmic bodies, called dwarfs.

The neutron matter. Pulsars.

Dwarfs may be considered as stars where a process of neutronization is just beginning. At a nuclear density, plasma turns into neutron matter. [1]

At taking into account the above assumptions, the stars composed of neutron matter, must also have the same equilibrium condition Eq.((11.9)). As the density of neutron matter:

$$n_n = \frac{p_F^3}{3\pi^2\hbar^3} = \frac{\xi^3}{3\pi^2}\left(\frac{m_n c}{\hbar}\right)^3 \tag{11.12}$$

(where m_n - the neutron mass), then the condition ((11.9)) allows to determine the equilibrium density of matter within a neutron star

$$n_n = n_n \approx 4 \cdot 10^{39} \tag{11.13}$$

particles in cm^3. The substitution of the values of the neutron density in the equilibrium condition Eq.(5.2) shows that, in accordance with our assessment of all the neutron stars in the steady state should have a mass of order of magnitude equal to the mass of the Sun. The measured mass distribution of pulsar composing binary stars [21] is shown on Fig.11.2. It can be considered as a confirmation of the last conclusion.

11.2.4 The hot relativistic plasma.Quasars

Plasma is hot if its temperature is higher than degeneration temperature of its electron gas. The ratio of plasma temperature in the core of a star to the temperature of degradation of its electron gas for case of non-relativistic hot star plasma is (Eq.(2.23))

$$\frac{\mathbb{T}_\star}{T_F(n_\star)} \approx 40 \tag{11.14}$$

it can be supposed that the same ratio must be characteristic for the case of a relativistic hot star. At this temperature, the radiation pressure plays a main role and accordingly the equation of the pressure balance takes the form:

$$\frac{GM^2}{6RV} \approx \frac{\pi^2}{45}\frac{(k\mathbb{T}_\star)^4}{(\hbar c)^3} \approx \left(\frac{\mathbb{T}_\star}{T_F}\right)^3 kTn \tag{11.15}$$

[1] At nuclear density neutrons and protons are indistinguishable inside pulsars as inside a huge nucleus. It permits to suppose a possibility of gravity induced electric polarization in this matter.

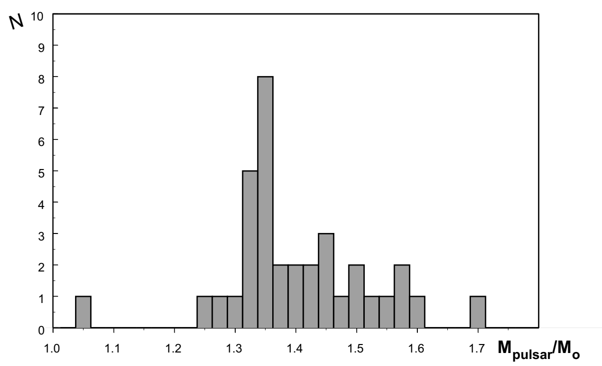

Figure 11.2: The mass distribution of pulsars from binary systems [21].On abscissa the logarithm of pulsar mass in solar mass is shown.

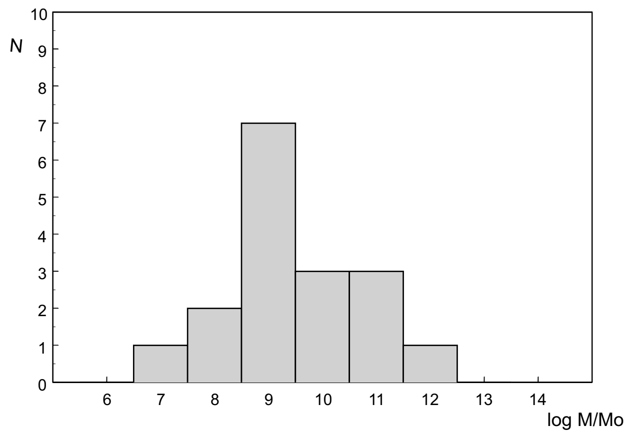

Figure 11.3: The mass distribution of galaxies. [4]. On the abscissa, the logarithm of the galaxy mass over the Sun mass is shown.

This makes it possible to estimate the mass of a hot relativistic star

$$M \approx \left(\frac{T_\star}{T_F}\right)^6 \left(\frac{\hbar c}{G m_p^2}\right)^{3/2} m_p \approx 10^9 M_\odot \tag{11.16}$$

According to the existing knowledge, among compact celestial objects only quasars have masses of this level. Apparently it is an agreed-upon opinion that quasars represent some relatively short stage of evolution of galaxies. If we adhere to this hypothesis, the lack of information about quasar mass distribution can be replaced by the distribution of masses of galaxies [4](Fig.11.3). It can be seen, that this distribution is in a qualitative agreement with supposition that quasars are composed from the relativistic hot plasma.

Certainly the used estimation Eq.(11.14) is very arbitrary. It is possible to expect the existing of quasars which can have substantially lesser masses.

As the steady-state particle density of relativistic matter n_r is known (Eq.(11.10)), one can estimate the quasar radius:

$$R_{qu} \approx \sqrt[3]{\frac{M_{qu}}{n_r m_p}} \approx 10^{12} cm. \tag{11.17}$$

It is in agreement with the astronomer measuring data obtained from periods of the their luminosity changing.

11.2.5 About the cosmic object categorizations

Thus, it seems possible under some assumptions to find characteristic parameters of different classes of stars, if to proceed from EOS of atomic, plasma and neutron substances. The EOS types can be compared with classes of celestial bodies. As any other EOS are unknown, it gives a reason to think that all classes of celestial bodies are discovered (Fig.11.4).

11.3 Several words about star evolution

There are not formulas in this section, which could serve as a handhold for suggestions. The formulas of the previous sections can not help too in understanding that, as the evolution of the stars can proceed and as their transitions them from the one class to another are realized, since these formulas were received for the description of the star steady-state. There the comparison of the schemes to the substance categorization (Fig.(11.1)) and the categorization of star classes (Fig.(11.4)) can serve as the base of next suggestions.

At analyzing of these schemes, one can suppose that the evolution of stellar objects is going at the reduction of their temperature. In light of it, it is possible to suppose the existing of one more body, from which the development began. Really, neutron matter at nuclear density (Eq.(11.13)) is not ultra-relativistic. At this density, corresponding

	Low T	High T
Atomic substance	asteroid	gaint
Plasma $p_F \ll m_e c$	planet	star
Plasma $p_F \approx m_e c$	dwarf	quasar
neutron matter $p_F \approx m_n c$	pulsar	?

Figure 11.4: The categorization of celestial bodies

to $p_F \approx mc$, its energy and pressure can depend on the temperature if this temperature it is high enough.[2] It seems, there is no thermodynamical prohibition to imagine this matter so hot when the neutron gas is non-degenerate. An estimation shows that it can be possible if mass of this body is near to $10^{50}g$ or even to $10^{55}g$. As it is accepted to think that full mass of the Universe is about $10^{53}g$, it can be assumed that on an early stage of development of Universe, there was some celestial body with the mass about $10^{53}g$ composed by the neutron matter at the approximately nuclear density with at the temperature above $10^{12}K$. After some time, with temperature decreased it has lost its stability and decayed into quasars with mass up to $10^{12}M_{Ch}$, consisting of the non-degenerate relativistic plasma at $T > 10^{10}K$. After next cooling at loosing of stability they was decaying on galaxies of hot stars with mass about $M \approx M_{Ch}$ and core temperature about $T \approx 10^7K$, composed by non-relativistic hot plasma. A next cooling must leads hot stars to decaying on dwarfs, pulsars, planets or may be on small bodies. The substances of these bodies (in their cores) consists of degenerate plasma (degenerate electron subsystem and cold nuclear subsystem) or cold neutron matter, it makes them stable in expanding and cooling Universe.[3] It is important to emphasize, that the gravity induced electric polarization excludes the possibility of the gravitational collapse as the last stage of the star evolutions.

11.4 About ≪black holes≫

It seems that the idea about the ≪black holes≫ existence is organic related to the suggestion about an inevitable collapse of large cosmic bodies on the last stage of their evolutions. However, the models of collapsing masses were appearing as a consequence of the rejection from attention of the gravity induced electric polarization of the intra-stellar plasma, which was considered in previous chapters. If to take into account this mechanism, the possibility of collapse must be excludes. It allows newly to take a look on the ≪black hole≫ problem.

In accordance with the standard approach, the Schwarzshield radius of ≪black hole≫ with M is

$$r_{bh} = \frac{2GM}{c^2} \tag{11.18}$$

and accordingly the average density of ≪black holes≫:

$$\gamma_{bh} = \frac{3c^6}{32\pi M^2 G^3}. \tag{11.19}$$

The estimations which was made in previous chapters are showing that all large inwardly-galactic objects of all classes - a stars, dwarves, pulsars, giants - possess the mass of the order M_{Ch} (or $10M_{Ch}$). As the density of these objects are small relatively

[2] The ultra-relativistic matter with $p_F \gg mc$ is possessed by limiting pressure which is not depending on temperature.

[3] The temperature of plasma inside these bodies can be really quite high as electron gas into dwarfs, for example, will be degenerate even at temperature $T \approx 10^9K$.

to the limit (11.19). As result, a searching of ≪black hole≫ inside stellar objects of our Galaxy seems as hopeless.

On the other hand, stellar objects, consisting of hot relativistic plasma - a quasars, in accordance with their mass and density, may stay ≪black holes≫. The process of collapse is not needed for their creation. As the quasar mass $M_{qu} \gg M_{Ch}$, all other stellar objects must organize their moving around it and one can suppose that a ≪black hole≫ can exist at the center of our Galaxy.

Chapter 12

The Theory of Earth

12.1 The introduction

Seismic measurements show that as the depth grows, the density of the Earth's matter increases at first more or less monotonically. However, on the core-mantle interface it rises abruptly. There exist a series of Earth models [23],[24] for the description of the density growth, for the explanation of the reason of the separation of the Earth matter into the core and the mantle and the determination of the density jump on their interface. Quite separately there exist models for the explanation of the origin of the terrestrial magnetism. Among these models, there is one based on the hydrodynamic hypothesis. It was developed in the 1940's-1950's. At present it is generally adopted. All these models try to decide the basic problem - to obtain the answer: why the main magnetic field of the Earth near the poles is of the order of 1 Oe? Such statement of the basic problem of terrestrial magnetism models nowadays is unacceptable. Space flights, started in 1960's, and the further development of astronomy have allowed scientists to obtain data on magnetic fields of all planets of Solar system, as well as some their satellites and a number of stars. As a result, a remarkable and earlier unknown fact has been discovered. It appears that the magnetic moments of all space bodies (those which have been measured) are proportional to their angular momenta. The proportionality coefficient is approximately equal to $G^{1/2}/c$, where G is the gravitational constant, c is the velocity of light (fig.7.1).

Amazing is that this dependence remains linear within 20 orders of magnitude! This fact makes it necessary to reformulate the main task of the model of terrestrial magnetism. It should explain, first, why the magnetic moment of the Earth, as well as of other space bodies, is proportional to its angular momentum and, second, why the proportionality coefficient is close to the above given ratio of world constants. The aim of this paper is to show that it is possible to construct the theory of the Earth using the full energy minimization method. This theory allows us to apply a new approach to the separation of the Earth onto the core and the mantle and avoid the shortcomings of previous models connected with their independent existence.

12.2 Equation of state

First, to create a theory of the Earth, we need to find the radial dependence of the terrestrial matter density $\gamma(r)$. To do this, it is necessary to write the equation of equilibrium for forces applied to the matter and the state equation of matter, i.e. the dependence of the matter density on the pressure. It is assumed that at small pressures the dependence of the matter density $\gamma(r)$ on the pressure p is described by Hook's law:

$$\gamma = \frac{\gamma_0}{1 - \frac{p}{B}} \tag{12.1}$$

i.e. at small pressures the equation of state is

$$p = B\left(1 - \frac{\gamma_0}{\gamma}\right), \tag{12.2}$$

where γ_0 is the matter density at zero pressure; B is the bulk module of matter. At high pressures the bulk module itself starts to depend on the density. This dependence can be described by the polytropic function

$$B = \alpha\gamma^{1+1/k}. \tag{12.3}$$

where α is a constant, k is a polytropic index describing the elastic property of matter ($k = 0$ describes incompressible matter). Thus, the equation of state can be written as

$$p = \alpha\gamma^{1+1/k}\left(1 - \frac{\gamma_0}{\gamma}\right). \tag{12.4}$$

At small pressures it transforms into Hook's law and at higher pressures it transforms into the standard polytropic equation

$$p = \alpha\gamma^{1+1/k}. \tag{12.5}$$

12.3 The core and the mantle

Let us assume that the considered spherical body (the Earth or any other planet) is divided into two regions - an inner core and an outer mantle. Thus, we shall assume that the mantle is composed of hard rock of the basalt type and is characterized, as generally accepted, by a polytropic index $k = 1$.

Under action of ultrahigh pressure, atoms in the core lose their outer electron shell to reduce their volume and form dense plasma. In this state, substance is characterized by the polytropic index $k = 3/2$. Plasma is electrically polarized matter and we can assume that the matter inside the core may be electrically polarized by gravity if it is energetically favorable.

As a rule, this possibility is not considered at all on the basis of the fact that the electrical polarization is connected with the appearance of some additional energy and is, therefore, energetically disadvantageous. At the same time, it escapes completely everybody's attention, that the electrical polarization changes and even can reduce other types of energy, such as gravitational and inner energy. Assuming that the core of the planet can be electrically polarized, we shall have as a purpose of our solution the determination of its radius R_n for the minimum of its full energy. If this configuration corresponds to the body with zero R_n, it will mean that the separation of the planet into an electrically polarized core and an unpolarized mantle is energetically disadvantageous. In view of mechanical strains, we shall assume that the planet matter has a homogeneous chemical composition with the density γ_0 and the bulk module B_0 at zero pressure. We shall assume that the electrical polarization intensity $\overrightarrow{P}$ inside a core is proportional to gravity

$$\overrightarrow{P} = (4\pi G^{1/2})^{-1}\overrightarrow{g}. \tag{12.6}$$

Thus, inside the core the effect of gravitation is completely compensated by the electric force

$$\gamma_n\overrightarrow{g} + \rho\overrightarrow{E} = 0, \tag{12.7}$$

where γ_n is the density of matter inside the core, g is the gravity acceleration, $\rho = -div\overrightarrow{P}$ is the charge density connected with polarization, $\overrightarrow{E} = -4\pi\overrightarrow{P}$ is the electric field strength connected with polarization. This becomes possible because the behavior of gravitational and electric field intensities has similar descriptions:

$$\begin{aligned} div\overrightarrow{E} &= 4\pi\rho \\ div\overrightarrow{g} &= -4\pi G\gamma_n. \end{aligned} \tag{12.8}$$

Due to such an electrical polarization distribution a bounded volume electric charge exists inside the core and on its surface there is a surface charge of the opposite sign so that the total electric charge of the core is equal to zero. It is easy to see that the polarization jump on the core surface is immediately accompanied with a pressure jump or, speaking in terms of bounded charges, the surface charge tends to compress the charged core. Thus, although the effect of gravity in the core is compensated, its matter experiences the pressure of the entire mass over its surface

$$p_m(R_n) = \alpha_m\gamma_m^2(R_n)\left(1 - \frac{\gamma_0}{\gamma(R_n)}\right) \tag{12.9}$$

(R_n indicates that its value is taken on the core surface) and the pressure of the surface charge [3]:

$$p_e = \frac{2\pi}{9}G\gamma_n^2R_n^2. \tag{12.10}$$

This additional compression has a significant value, which is assumed to be sufficient to transform the core matter into the plasma state. In this case, the polarization sign is evident as soon as in this process the core matter acquires a positive charge while electrons are pushed out to the core surface. Estimations show that since the force of gravitation is weak compared to the electric force, the charge related to each ion is only about 10^{-15} of the electron charge (for the case of terrestrial gravitation).

Since the compressibility of dense plasma is determined by the bulk module of Fermi gas, the polytropic index of such a matter is 3/2. In this case, the pressures inside the core and the core density are constant,

$$p_n = \alpha_n \gamma_n^{5/3}\left(1 - \frac{\gamma_0}{\gamma_n}\right) = \alpha_m \gamma_m^2(R_n)\left(1 - \frac{\gamma_0}{\gamma_m(R_n)}\right) + \frac{2\pi}{9} G \gamma_n^2 R_n^2. \tag{12.11}$$

As a result, the density inside the core is higher than the one that would exist inside the planet in the absence of polarization. The equilibrium state of the mantle matter is described by

$$\frac{dp}{dr} = -\frac{G\gamma(r)M(r)}{r^2}, \tag{12.12}$$

where $M(r)$ is the mass of the matter confined to the radius r:

$$M(r) = 4\pi \int_0^r \gamma(r) r^2 dr. \tag{12.13}$$

Thus, from Eq.(12.12) for the mantle matter, we have

$$\gamma_m^2\left(1 - \frac{\gamma_0}{\gamma_m}\right) = A_m \int_r^R \gamma_m \left(\gamma_n \frac{R_n^3}{3} + \int_{R_n}^r \gamma_m x^2 dx\right)\frac{dx}{x^2}, \tag{12.14}$$

where $A_m = 4\pi G R_0^2 \gamma_0^2/B = 4\pi G R_0^2/\alpha_M$ and R_0 is the radius of the planet in the case when it is composed of incompressible matter . Under strain the full mass of the planet is naturally conserved

$$\gamma_n \frac{4\pi}{3} R_n^3 + 4\pi \int_{R_n}^R \gamma_m r^2 dr = \frac{4\pi}{3}\gamma_0 R_0^3. \tag{12.15}$$

Solving together Eqs.(12.11), (12.14), and (12.15), we find γ_n, γ_m and the ratio R/R_0 as functions of R_n.

12.4 The energy of a planet

Next, we have to answer the principal question of whether the existence of an electrically polarized core is energetically advantageous. The gravitational energy of the spherical body under the definition is

$$\varepsilon_g = -G \int_0^R \frac{M(r)}{r} dM(r). \tag{12.16}$$

It can be found for the known density distribution inside a planet $\gamma(r)$. Furthermore, accounting to the thermodynamic equation for chemical potential

$$d\chi = \frac{m'}{\gamma} dp \tag{12.17}$$

(where m' is the ion mass) from the equation of state Eq.(12.4), the chemical potential is

$$\chi = \alpha m' \left((k+1)\gamma^{1/k} - \frac{\gamma^{1/k} - k^2\gamma}{1-k} \right) \tag{12.18}$$

and the density of the internal energy of the core is

$$\varepsilon_{in} = \frac{\chi\gamma}{m'} - p = \alpha_n \left(\frac{3}{2}\gamma_n^{5/3} + 3\gamma_n^{2/3} - \frac{9}{2}\gamma_n \right). \tag{12.19}$$

Doing analogous calculations for the mantle, we obtain

$$\varepsilon_{im} = \alpha_m \left(\frac{\gamma_m^2(r)}{\gamma_0^2} + \frac{\gamma_m(r)}{\gamma_0} - \frac{\gamma_m(r)}{\gamma_0} ln \frac{\gamma_m(r)}{\gamma_0} - 2 \right). \tag{12.20}$$

The electric energy exists only inside the core and its density is

$$\frac{E^2(r)}{8\pi} = \frac{2\pi}{9} G\gamma_n^2 r^2. \tag{12.21}$$

Since the thermal energy is neglected, to calculate the full energy of the planet, it is necessary to integrate Eqs.(12.19), (12.20), and (12.22) over the volume of the planet and sum them and Eq.(12.16). To do this, we need to determine the values of constants composing these equations.

12.5 The density distribution inside the Earth

The mass M and radius R of the Earth are known. Therefore, we know the average density of the Earth $< \gamma > \cong 5.5 g/cm^3$. On the basis of the geophysical data, we accept that the density of matter and bulk module on the surface of the mantle is $\gamma_0 \cong 3.2 g/cm^3$ and $B = 1.3 \cdot 10^{12} dyn/cm^2$. These values are characteristic for basalts [24]. Based on the above said we determine R_0 and the parameter α_m. We can found the value of the parameter α_m as we know the values of γ_0 and $< \gamma >$ and therefore we can find the ratio

$$\frac{R}{R_0} = \left(\frac{\gamma_0}{< \gamma >} \right)^{1/3} = 0.835. \tag{12.22}$$

Next from all possible solutions we choose the one that actually meets the condition (12.22). In fact this procedure is reduced to choosing the parameter m', i.e. the ion mass related to each free electron in electron-ion plasma of the core. The total energy (related to GM/R_0) is plotted as a function of the parameter m' in fig.(12.1).

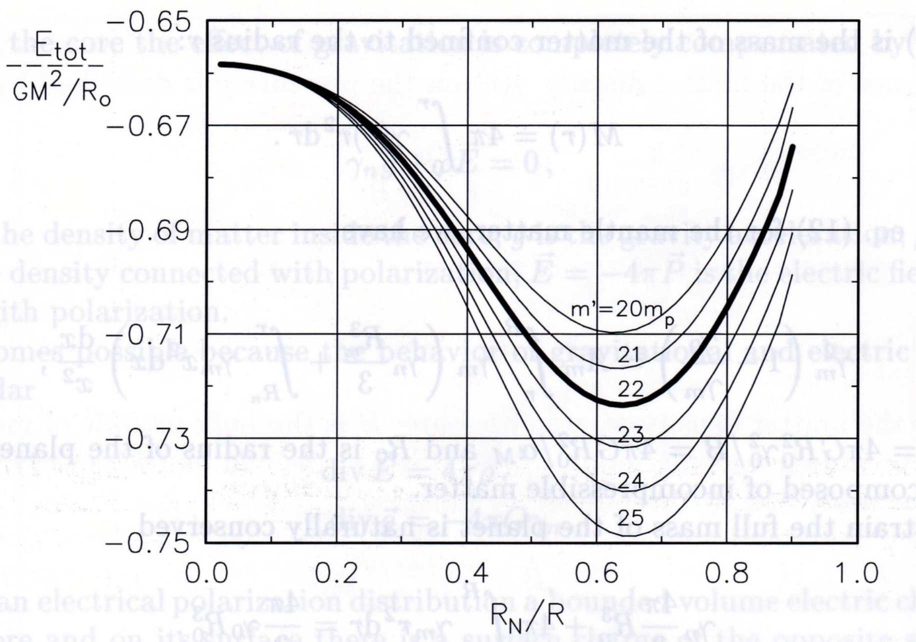

Figure 12.1: The dependence of the total energy of the planet (over $\frac{GM^2}{R_0}$) on the size of the polarized core composed of some metal with different averaged ion mass m' per one conductivity electron.

The dependence of the outer radius of the planet R (related to R_0) over the core radius R_n at different values of m' is shown in fig.(12.2). It can be seen that of the whole family only the curve obtained on the condition that in the core there exist approximately 22 nucleons per electron of electron-ion plasma satisfies to the ratio $\frac{R}{R_0} = 0.835$ (Eq.(12.22)) for $R_n/R = 0.65$.

Finally, knowing m' and R/R_0, we can find the distribution of the density of matter inside the planet. This is illustrated in fig.12.2 for $m' = 22m_p$ (m_p is the proton mass) and $R_n/R = 0.65$.

Thus, the calculation shows that for the Earth it is energy advantageous to have a electrically polarized core. Radius R_n and density γ_n of the core are approximately equal to $4 \cdot 10^3$km and $10g/cm^3$, respectively. On the mantle-core interface, the matter density drops sharply to $5g/cm^3$ and then it decreases almost linearly as the radius increases. The measured dependence of the matter density inside the Earth is shown also in fig.12.3. It is determined by measuring the propagation velocity of seismic waves. Being different, the calculated and measured dependencies coincide in the principal feature, i.e. they both indicate the existence of approximately equal jumps of the density on the core-mantle interface at about the half-radius of the planet.

12.6 The moment of inertia and the magnetic moment of the Earth

Knowing the matter distribution between the core and the mantle and their sizes, it is possible to calculate the moment of inertia for our theory. For a spherical body with a radial density distribution, we have

$$I = \frac{8\pi}{3} \int_0^r \gamma(r) r^4 dr. \tag{12.23}$$

In our case, we obtain

$$\frac{I}{MR^2} = 0.339. \tag{12.24}$$

It is in good agreement with the measured value 0.331.

It is obvious that the most bright and important result of the developed theory is the understanding of the mechanism of the generation of the terrestrial magnetic field. It is very simple: the rotation of the electrically polarized core (together with the planet) about its axis with the frequency Ω produces the magnetic moment

$$\mu = \frac{8\pi}{45c} G^{1/2} \Omega \gamma_n R_n^5. \tag{12.25}$$

Substituting appropriate values, we obtain $\mu \cong 4 \cdot 10^{25} Gs/cm^3$ which is almost exactly equal to one-half of the observed value of the moment $8.05 \cdot 10^{25} Gs/cm^3$.

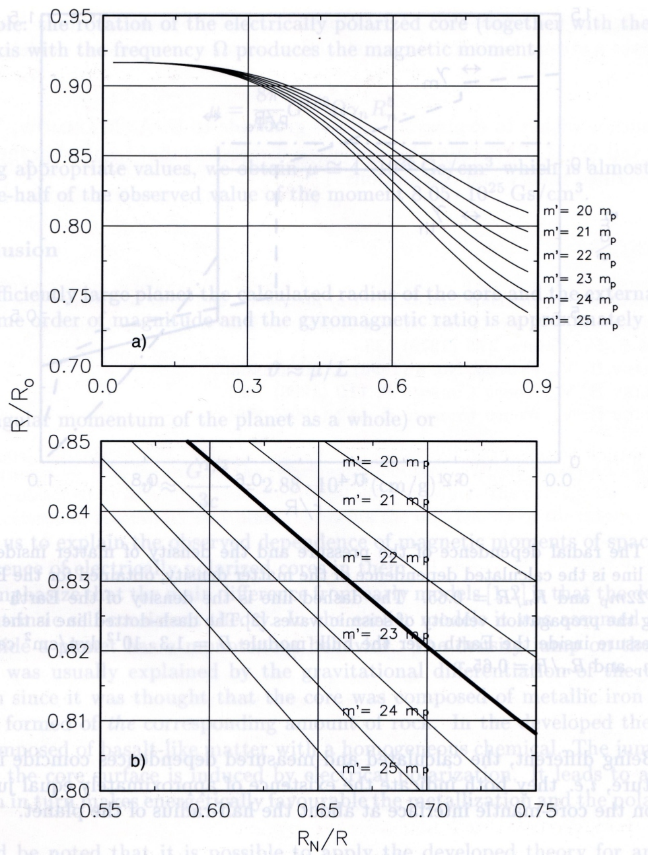

Figure 12.2: a) The external radius of the planet R (over R_0) vs. the size of the core R_n/R. b) The same dependence to larger scale.

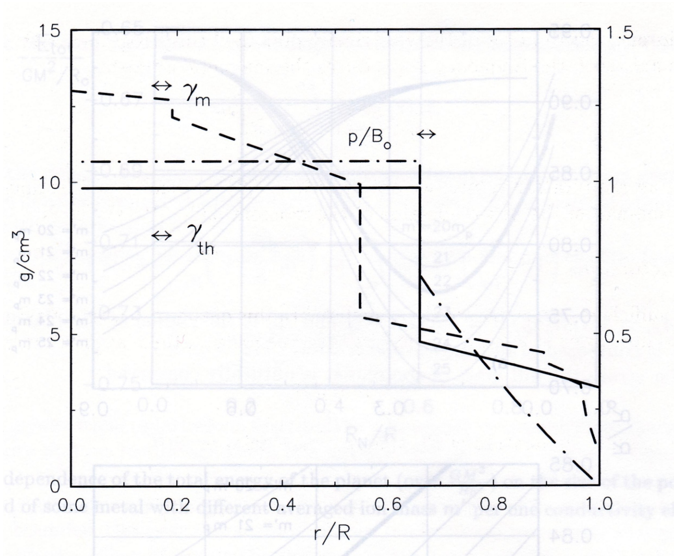

Figure 12.3: The radial dependence of the pressure and the density of matter inside the Earth. The solid line is the calculated dependence of the matter density, obtained for the Earth theory at $m' = 22m_p$ and $R_n/R = 0.65$. The dashed line is the density of the Earth obtained by measuring the propagation velocity of seismic waves [24]. The dash-dotted fine is the dependence of the pressure inside the Earth over the bulk module $B = 1.3{\cdot}10^{12} dyn/cm^2$ calculated for $m' = 22m_p$ and $R_n/R = 0.65$.

Chapter 13

The conclusion

Evidently, the main conclusion from the above consideration consists in statement of the fact that now there are quite enough measuring data to place the theoretical astrophysics on a reliable foundation. All above measuring data are known for a relatively long time. The traditional system of view based on the Euler equation in the form (1.1) could not give a possibility to explain and even to consider. with due proper attention, to these data. Taking into account the gravity induced electric polarization of plasma and a change starting postulate gives a possibility to obtain results for explanation of measuring data considered above.

Basically these results are the following.

Using the standard method of plasma description leads to the conclusion that at conditions characteristic for the central stellar region, the plasma has the minimum energy at constant density $n_\star$ (Eq.(2.18)) and at the constant temperature $\mathbb{T}_\star$ (Eq.(5.21)).

This plasma forms the core of a star, where the pressure is constant and gravity action is balanced by the force of the gravity induced by the electric polarization. The virial theorem gives a possibility to calculate the stellar core mass $\mathbb{M}_\star$ (Eq.(5.25)) and its radius $\mathbb{R}_\star$ (5.27). At that the stellar core volume is approximately equal to 1/1000 part of full volume of a star.

The remaining mass of a star located over the core has a density approximately thousand times smaller and it is convenient to name it a star atmosphere. At using thermodynamical arguments, it is possible to obtain the radial dependence of plasma density inside the atmosphere $n_a \approx r^{-6}$ (Eq.(4.17)) and the radial dependence of its temperature $\mathbb{T}_a \approx r^{-4}$ (Eq.(4.18)).

It gives a possibility to conclude that the mass of the stellar atmosphere $\mathbb{M}_a$ (Eq.(4.19)) is almost exactly equal to the stellar core mass.Thus, the full stellar mass can be calculated. It depends on the ratio of the mass and the charge of nuclei composing the plasma. This claim is in a good agreement with the measuring data of the mass distribution of both - binary stars and close binary stars (Fig.(5.1)-(5.2)) [1]. At that it is important that the upper limit of masses of both - binary stars and close binary stars - is in accordance with the calculated value of the mass of the hydrogen star (Eq.(5.26)). The obtained formula explains the origin of sharp peaks of stellar mass distribution - they evidence that the substance of these stars have a certain value of the ratio A/Z. In particular the solar plasma according to (Eq.(5.26)) consists of nuclei with $A/Z = 5$.

Knowing temperature and substance density on the core and knowing their radial dependencies, it is possible to estimate the surface temperature $\mathbb{T}_0$ (5.38) and the radius of a star $\mathbb{R}_0$ (5.37). It turns out that these measured parameters must be related to the star mass with the ratio $\mathbb{T}_0\mathbb{R}_0 \sim \mathbb{M}^{5/4}$ (5.46). It is in a good agreement with measuring data (Fig.(5.3)).

Using another thermodynamical relation - the Poisson's adiabat - gives a way to determine the relation between radii of stars and their masses $\mathbb{R}_0^3 \sim \mathbb{M}^2$ (Eq.(6.16)), and between their surface temperatures and masses $\mathbb{T}_0 \sim \mathbb{M}^{5/7}$ (Eq.(6.20)). It gives the quantitative explanation of the mass-luminosity dependence (Fig.(6.3)).

According to another familiar Blackett's dependence, the giromagnetic ratios of celestial bodies are approximately equal to $\sqrt{G}/c$. It has a simple explanation too. When there is the gravity induced electric polarization of a substance of a celestial body, its rotation must induce a magnetic field (Fig.(7.1)). It is important that all (composed by eN-plasma) celestial bodies - planets, stars, pulsars - obey the Blackett's dependence. It confirms a consideration that the gravity induced electric polarization must be characterizing for all kind of plasma. The calculation of magnetic fields of hot stars shows that they must be proportional to rotation velocity of stars (7.11). Magnetic fields of Ap-stars are measured, and they can be compared with periods of changing of luminosity of these stars. It is possible that this mechanism is characteristic for stars with rapid rotation (Fig.(7.2)), but obviously there are other unaccounted factors.

Taking into account the gravity induced electric polarization and coming from the Clairault's theory, we can describe the periastron rotation of binary stars as effect descended from non-spherical forms of star cores. It gives the quantitative explanation of this effect, which is in a good agreement with measuring data (Fig.(8.1)).

The solar oscillations can be considered as elastic vibrations of the solar core. It

[1] The measurement of parameters of these stars has a satisfactory accuracy only.

permits to obtain two basic frequencies of this oscillation: the basic frequency of sound radial oscillation of the core and the frequency of splitting depending on oscillations of substance density near its equilibrium value (Fig.(9.2)).

The plasma can exists in four possible states. The non-relativistic electron gas of plasma can be degenerate and non-degenerate. Plasma with relativistic electron gas can have a cold and a hot nuclear subsystem. Together with the atomic substance and neutron substance, it gives seven possible states. It suggests a way of a possible classification of celestial bodies. The advantage of this method of classification is in the possibility to estimate theoretically main parameters characterizing the celestial bodies of each class. And these predicted parameters are in agreement with astronomical observations. It can be supposed hypothetically that cosmologic transitions between these classes go in direction of their temperature being lowered. But these suppositions have no formal base at all.

Discussing formulas obtained, which describe star properties, one can note a important moment: these considerations permit to look at this problem from different points of view. On the one hand, the series of conclusions follows from existence of spectrum of star mass (Fig.(5.1)) and from known chemical composition dependence. On the another hand, the calculation of natural frequencies of the solar core gives a different approach to a problem of chemical composition determination. It is important that for the Sun, both these approaches lead to the same conclusion independently and unambiguously. It gives a confidence in reliability of obtained results.

In fact, the calculation of magnetic fields of hot stars agrees with measuring data on order of the value only. But one must not expect the best agreement in this case because calculations were made for the case of a spherically symmetric model and measuring data are obtained for stars where this symmetry is obviously violated. But it is important, that the all remaining measuring data (all known of today) confirm both - the new postulate and formulas based on it. At that, the main stellar parameters - masses, radii and temperatures - are expressed through combinations of world constants and at that they show a good accordance with observation data. It is important that quite a satisfactory quantitative agreement of obtained results and measuring data can be achieved by simple and clear physical methods without use of any fitting parameter. It gives a special charm and attractiveness to star physics.

For a sufficiently large planet the calculated radius of the core and the external radius have the same order of magnitude and the gyromagnetic ratio is approximately equal to

$$\vartheta = \frac{\mu}{L} \tag{13.1}$$

(L is the angular momentum of the planet as a whole) or

$$\vartheta \approx \frac{G^{1/2}}{3c} \approx 2.88 \cdot 10^{-15} (cm/g)^{1/2} \tag{13.2}$$

This allows us to explain the observed dependence of magnetic moments of space bodies by the existence of electrically polarized cores in them. Let us emphasize that the main difference from early models [23],[24] is that the developed one is an intrinsic self-consistent theory. In the earlier models it was assumed that the pressure inside a planet has a monotonous behavior. The density jump on the surface of the core was usually explained by the gravitational differentiation of the chemical composition since it was thought that the core was composed of metallic iron and the mantle was formed of the corresponding amount of rock. In the developed theory, the planet is composed of basalt-like matter with a homogeneous chemical. The jump of the pressure on the core surface is induced by electrical polarization. It leads to a density jump, which in turn makes energetically favorable the polarization of the core. It should be noted that it is possible to apply the developed theory for any space body with a sufficiently large mass. Actually the condition for the appearance of electrical polarization inside the planet can be reduced to the requirement that it has a sufficiently large mass. If a space body has the density and the bulk module characteristic for the Earth, its mass must be larger than 10^{26} g. Thus, the existence of polarization is energetically disadvantageous in small bodies such as Moon and asteroids. It is also necessary to mention that the developed theory does not substitute the dynamo-model. It simply completes it with the mechanism of the creation of a bare field. This statement is supported by the fact that the calculated magnetic moment of the Earth is two times smaller than its measured magnetic moment and that the magnetic moments of a number of other planets have the same order of magnitude as Eq.(13.1) but the opposite sign. In conclusion, it is noted that the developed model is actually the theory of the Earth as it has no free parameters. In order to find the basic characteristics of the interior structure of the Earth, we use the numerical values of its mass and radius known unambiguously and the values of the density of matter and the bulk module on the mantle surface, whose were also not chosen arbitrarily.

Bibliography

[1] Vasiliev B.V.: The gravity-induced electric polarization of electron-nuclear plasma and related astrophysical effects *Nuovo Cimento B*, **116**, pp.617-634, (2001)

[2] Vasiliev B.V.: Why spontaneous electric polarization can arise inside cosmic bodies? *Nuovo Cimento B*, **112**, pp.1361-1372, (1997)

[3] Vasiliev B.V.: Can the existence of the magnetic moments of cosmic bodies be explained by internal spontaneous electric polarization?*Nuovo Cimento B*, **110**, pp.381389, (1996)

[4] Allen C.W. - Astrophysical quantities, 1955,University of London, The Athlone Press.

[5] Beskin V.S., Gurevich A.V., Istomin Ya.N.:
Physics of the Pulsar Magnetosphere (Cambridge University Press) (1993)

[6] Blackett P.M.S.: *Nature*, **159**, 658, (1947)

[7] Chandrasekhar S.: Monthly Notices of the RAS **93**, 449, (1933)

[8] Christensen-Dalsgaard, J.: Stellar oscillation, Institut for Fysik og Astronomi, Aarhus Universitet, Denmark, (2003)

[9] Elsworth, Y. at al. - In Proc. GONG'94 Helio- and Astero-seismology from Earth and Space, eds. Ulrich,R.K., Rhodes Jr,E.J. and Däppen,W., Asrtonomical Society of the Pasific Conference Series, vol.76, San Fransisco,**76**, 51-54.

[10] Heintz W.D.: *Double stars* In Geoph. and Astroph. monographs, **15**, D.Reidel Publ. Corp., (1978)

[11] Khaliullin K.F.: *Dissertation*, Sternberg Astronomical Institute, Moscow, (Russian)(2004) (see Table in Appendix on http://astro07.narod.ru)

[12] Landau L.D. and Lifshits E.M.: Statistical Physics, **1**, 3rd edition, Oxford:Pergamon, (1980)

[13] Landau L.D. and Lifshits E.M.: Electrodynamics of condensed matter, **1**, 3rd edition, Oxford:Pergamon, (1980)

[14] Landau L.D. and Lifshits E.M.: The Classical Theory of Fields. **1**, Pergamon Press, N.Y. (1971)

[15] Leung Y.C.: *Physics of Dense Matter* In Science Press/World Scientific, Beijing and Singapore, (1984)

[16] I.I.Romanyuk at al. Magnetic Fields of Chemically Peculiar and Related Stars, Proceedings of the International Conference (Nizhnij Arkhyz, Special Astrophysical Observatory of Russian Academy of Sciences, September 24-27, 1999), eds: Yu. V. Glagolevskij and I.I. Romanyuk, Moscow,2000, pp. 18-50.

[17] Russel H.N.: Monthly Notices of the RAS **88**, 642, (1928)

[18] Sirag S.-P.: *Nature*, **275**, 535, (1979)

[19] Solar Physics, **175/2**, (http://sohowww.nascom.nasa.gov/gallery/Helioseismology)

[20] BlochJ., Mintz H. : Kinetics and mechanisms of metal hydrides formation - a review. J. of Alloys and Compound. v.253-254, pp.529-541 (1997)

[21] Thorsett S.E. and Chakrabarty D.: *E-preprint: astro-ph/9803260*, (1998)

[22] Vasiliev B.V. and Luboshits V.L.: *Physics-Uspekhi*, **37**, 345, (1994)

[23] Gutenberg B., Internal Constitution of the Earth, 2nd edition (Dover, New York) 1951.

[24] Jeffreys H.: The Earth, 3rd edition (Cambrige University Press) 1952.

Appendix

The Table of main parameters of close binary stars

(cited on Kh.F.Khaliullin's dissertation,
Sternberg Astronomical Institute.
In Russian)

N	Name of star	U period of apsidal rotation, years	P period of ellipsoidal rotation, days	$\mathbb{M}_1/\mathbb{M}_\odot$ mass of component 1, in the Sun mass	$\mathbb{M}_2/\mathbb{M}_\odot$ mass of component 2, in the Sun mass	$\mathbb{R}_1/\mathbb{R}_\odot$ radius of 1 component in the Sun radius	$\mathbb{R}_2/\mathbb{R}_\odot$ radius of 2 component in the Sun radius	$\mathbb{T}_1$ temperature of 1 component, K	$\mathbb{T}_2$ temperature of 2 component, K	References
1	BW Aqr	5140	6.720	1.48	1.38	1.803	2.075	6100	6000	1,2
2	V 889 Aql	23200	11.121	2.40	2.20	2.028	1.826	9900	9400	3,4
3	V 539 Ara	150	3.169	6.24	5.31	4.512	3.425	17800	17000	5,12,24,67
4	AS Cam	2250	3.431	3.31	2.51	2.580	1.912	11500	10000	7,13
5	EM Car	42	3.414	22.80	21.40	9.350	8.348	33100	32400	8
6	GL Car	25	2.422	13.50	13.00	4.998	4.726	28800	28800	9
7	QX Car	361	4.478	9.27	8.48	4.292	4.054	23400	22400	10,11,12
8	AR Cas	922	6.066	6.70	1.90	4.591	1.808	18200	8700	14,15
9	IT Cas	404	3.897	1.40	1.40	1.616	1.644	6450	6400	84,85
10	OX Cas	40	2.489	7.20	6.30	4.690	4.543	23800	23000	16,17
11	PV Cas	91	1.750	2.79	2.79	2.264	2.264	11200	11200	18,19
12	KT Cen	260	4.130	5.30	5.00	4.028	3.745	16200	15800	20,21
13	V 346 Cen	321	6.322	11.80	8.40	8.263	4.190	23700	22400	20,22
14	CW Cep	45	2.729	11.60	11.10	5.392	4.954	26300	25700	23,24
15	EK Cep	4300	4.428	2.02	1.12	1.574	1.332	10000	6400	25,26,27,6
16	α Cr B	46000	17.360	2.58	0.92	3.314	0.955	9100	5400	28,29
17	Y Cyg	48	2.997	17.50	17.30	6.022	5.680	33100	32400	23,30
18	Y 380 Cyg	1550	12.426	14.30	8.00	17.080	4.300	20700	21600	31
19	V 453 Cyg	71	3.890	14.50	11.30	8.607	5.410	26600	26000	17,32,33
20	V 477 Cyg	351	2.347	1.79	1.35	1.567	1.269	8550	6500	34,35
21	V 478 Cyg	26	2.881	16.30	16.60	7.422	7.422	29800	29800	36,37
22	V 541 Cyg	40000	15.338	2.69	2.60	2.013	1.900	10900	10800	38,39
23	V 1143 Cyg	10300	7.641	1.39	1.35	1.440	1.226	6500	6400	40,41,42
24	V 1765 Cyg	1932	13.374	23.50	11.70	19.960	6.522	25700	25100	28
25	DI Her	29000	10.550	5.15	4.52	2.478	2.689	17000	15100	44,45,46,47
26	HS Her	92	1.637	4.25	1.49	2.709	1.485	15300	7700	48,49
27	CO Lac	44	1.542	3.13	2.75	2.533	2.128	11400	10900	50,51,52
28	GG Lup	101	1.850	4.12	2.51	2.644	1.917	14400	10500	17
29	RU Mon	348	3.585	3.60	3.33	2.554	2.291	12900	12600	54,55
30	GN Nor	500	5.703	2.50	2.50	4.591	4.591	7800	7800	56,57
31	U Oph	21	1.677	5.02	4.52	3.311	3.110	16400	15200	53,58,37
32	V 451 Oph	170	2.197	2.77	2.35	2.538	1.862	10900	9800	59,60
33	β Ori	228	5.732	19.80	7.50	14.160	8.072	26600	17800	61,62,63
34	FT Ori	481	3.150	2.50	2.30	1.890	1.799	10600	9500	64
35	AG Per	76	2.029	5.36	4.90	2.995	2.606	17000	17000	23,24
36	IQ Per	119	1.744	3.51	1.73	2.445	1.503	13300	8100	65,66
37	ζ Phe	44	1.670	3.93	2.55	2.851	1.852	14100	10500	11,67
38	KX Pup	170	2.147	2.50	1.80	2.333	1.593	10200	8100	21
39	NO Pup	37	1.257	2.88	1.50	2.028	1.419	11400	7000	11,69
40	VV Pyx	3200	4.596	2.10	2.10	2.167	2.167	8700	8700	70,71
41	YY Sgr	297	2.628	2.36	2.29	2.196	1.992	9300	9300	72
42	V 523 Sgr	203	2.324	2.10	1.90	2.682	1.839	8300	8300	73
43	V 526 Sgr	156	1.919	2.11	1.66	1.900	1.597	7600	7600	74
44	V 1647 Sgr	592	3.283	2.19	1.97	1.832	1.669	8900	8900	75
45	V 2283 Sgr	570	3.471	3.00	2.22	1.957	1.656	9800	9800	76,77
46	V 760 Sco	40	1.731	4.98	4.62	3.015	2.642	15800	15800	78
47	AO Vel	50	1.585	3.20	2.90	2.623	2.954	10700	10700	79
48	EO Vel	1600	5.330	3.21	2.77	3.145	3.284	10100	10100	21,63
49	α Vir	140	4.015	10.80	6.80	8.097	4.394	19000	19000	80,81,68
50	DR Vul	36	2.251	13.20	12.10	4.814	4.369	28000	28000	82,83

Bibliography

[1] Khaliulilin Kh.F. and Kozyreva V.S.
Apsidal motion in the eclipsing binary system of BW Aqr
Astrophys. and Space Sci., **120** (1986) 9-16.

[2] Imbert M.
Photoelectric radial velosities of eclipsing binaries. IV. Orbital elements of BW Aqr,
Astron.Astrophys.Suppl., **69** (1987) 397-401.

[3] Khaliulilin Kh.F. and Khaliulilina A.I.
Fotometricheskoe issledovanie zatmenno-dvoinoi sistemy s relativistskim vrasheniem orbity V889 Aql,
Astronom.zh.,**66**(1989)76-83 (in Russian).

[4] Khaliulilin Kh.F. and Khaliulilina A.I.
K probleme vrashenia linii apsid v zatmennoi sisteme V889 Aql,
Astron.cirk., **N1486** (1987) 5-7 (in Russian).

[5] Clausen J.V.
V 539 Arae: first accurate dimensions of eclipsing binaries,
Astron.Astrophys., **308** (1996) 151-169.

[6] Lavrov M.I. and Lavrova N.V.
Revisia elementov fotometricheskoi orbity EK Cep,
Astron.cirk. **971** (1977) 3-4 (in Russian).

[7] Khaliulilin Kh.F. and Kozyreva V.S. *Apsidal motion in the eclipsing binary AS Cam*,
Astrophys. and Space Sci., **120** (1994) 115-122.

[8] Andersen J. and Clausen J.V., *Absolute dimensions of eclipsing binaries.XV. EM Cainae*,
Astron.Astrophys. **213** (1989) 183-194.

[9] Gemenez A. and Clausen J.V.,
Four-color photometry of eclipsing binaries. XXIIA. Photometric elements and apsidal motion of GL Cainae,
Astron.Astrophys. **161** (1986) 275-286.

[10] Andersen J., Clausen J.V., Nordstrom B. and Reipurth B.,
Absolute dimensions of eclipsing binaries.I. The early-type detached system QX Cainae,
Astron.Astrophys. **121** (1983) 271-280.

[11] Gemenez A., Clausen J.V. and Jensen K.S.
Four-color photometry of eclipsing binaries. XXIV. Aspidal motion of QX Cainae, ξ Phoenicis and NO Puppis,
Astron.Astrophys. **159** (1986) 157-165.

[12] De Greve J.P.
Evolutionary models for detached close binaries: the system Arae and QX Cainae,
Astron.Astrophys. **213** (1989) 195-203.

[13] Malony F.P., Guinan E.F. and Mukherjec J.
Eclipsing binary star as test of gravity theories
Astron.J. **102** (1991) 256-261.

[14] Mossakovskaya L.V.
New photometric elements of AR Cas, an eclipsing binary system with apsidal motion
Astron. and Astroph. Trans. **2** (1992) 163-167.

[15] Haffer C.M. and Collins G.M.
Computation of elements of eclipsing binary stars by high-speed computing machines
Astroph.J.Suppl., **7** (1962) 351-410.

[16] Crinklaw G. and Etzel P.
A photometric analisis of the eclipsing binary OX Cassiopeiae
Astron.J. **98** (1989) 1418-1426.

[17] Claret A. and Gimenez A.
The aspidal motion test of the internal stellar structure: comparision between theory and observations
Astron.Astroph. **277** (1993) 487-502.

[18] Wolf M.
Aspidal motion in the eclipsing binary PV Cassiopeiae
Monthly Not.Roy.Soc. **286** (1995) 875-878.

[19] Popper D.M.
Rediscussion of eclipsing binaries.XVII.The detached early A type binaries PV Cassiopeae and WX Cephei
Astron.J. **93** (1987) 672-677.

[20] Lavrov M.I. and Lavrova N.V.
Revisia fotometrichestih elementov u zatmennyh dvoinyh sistem s ekscentricheskimi orbitami.2.KT Cen
Trudy Kaz.Gor.AO **49** (1985) 18-24 (in Russian).

[21] Soderhjelm S.
Observations of six southern eclipsing binaries for apsidal motion
Astron.Astroph.Suppl.Ser **22** (1975) 263-283.

[22] Gemenez A., Clausen J.V. and Anderson J.
Four-color photometry of eclipsing binaries. XXIA. Photometric analysis and aspidal motion study of V346 Centauri,
Astron.Astrophys. **160** (1986) 310-320.

[23] Gemenez A., Chun-Hwey Kim and Il-Seong Nha
Aspidal motion in the early-type eclipsing binaries CW Cephei, Y Cyg and AG Per
Montly .Not.Roy.Astron.Soc. **224** (1987) 543-555.

[24] Bocula R.A.
Peresmotr elementov fotometricheskoi orbity zatmennyh sistem CW Cep, V 539 Ara, AG Per, AR Aur, RS Cha, ZZ Boo.
Peremennye zvezdy**21** (1983) 851-859 (in Russian).

[25] Khaliulilin Kh.F.
Relativistskoe vrashenie orbity zatmennoi dvoinoi sistemy EK Cep
Astron.zh.**60** (1983) 72-82 (in Russian).

[26] Tomkin J.
Secondaries of eclipsing binary. V. EK Cephei
Astroph.J. **271** (1983) 717-724.

[27] Claret A., Gemenez A. and Martin E.L.
A test case of stellar evolution the eclipsing binary EK Cephei
Astron.Astroph. **302** (1995) 741-744.

[28] Volkov I.M.
The discovery of apsidal motion in the eclipsing binary system α Cr B
Inf.Bull.Var.Stars N3876,(1993) 1-2.

[29] Quiroga R.J., van L.P.R.
Angular momenta in binary systems
Astroph.Space Sci. **146** (1988) 99-137.

[30] Hill G. and Holmgern D.E.
Studies of early-type varieble stars
Asrton.Astroph.**297** (1995) 127-134.

[31] Hill G. and Batten A.H.
Studies of early-type varieble stars.III. The orbit and physical dimensions for V 380 Cygni
Asrton.Astroph.**141** (1984) 39-48.

[32] Zakirov M.M.
Ob apsidalnom dvizhenii v dvoinoi sisteme V 453 Cyg
Astron.cirk.N1537,21 (in Russian).

[33] Karetnikov V.G.
Spectral investigation of eclipsing binary stars at the stage of mass exchange
Publ.Astron.Inst.Czech.**70** (1987) 273-276.

[34] Mossakovskaya L.V. and Khaliulilin Kh.F.
Prichina anomalnogo apsidalnogo dvizhenia v sisteme V 477 Cyg
Astron.cirk.N1536, 23-24 (in Russian).

[35] Gemenez A. and Quintana J.M.
Apsidal motion and revised photometry elements of the eccentric eclipsing binary V 477 Cyg
Astron.Astrophys. **260** (1992) 227-236.

[36] Mossakovskaya L.V. and Khaliulilin Kh.F.
Vrashenie linii apsid v sisteme V 478 Cyg
Pisma v Astron.zh.**22** (1996) 149-152.

[37] Popper D.M. and Hill G.
Rediscussion of eclipsing binaries.XVII.Spectroscopic orbit of OB system with a cross-correlation procedure
Astron.J. **101** (1991) 600-615.

[38] Khaliulilin Kh.F.
The unique eclipsing binary system V 541 Cygni with relativistic apsidal motion
Astrophys.J. **229** (1985) 668-673.

[39] Lines R.D.,Lines H., Guinan E.F. and Carroll
Time of minimum determination of eclipsing binary V 541 Cygni
Inf.Bull.Var.Stars N3286,1-3.

[40] Khaliulilin Kh.F.
Vrashenie linii apsid v zatmennoi sisteme V 1143 Cyg
Asrton. cirk.N1262,1-3 (in Russian).

[41] Andersen J., Garcia J.M., Gimenes A. and Nordstom B.
Absolute dimension of eclipsing binaries.X. V1143 Cyg
Astron.Astrophys. **174** (1987) 107-115.

[42] Burns J.F., Guinan E.F. and Marshall J.J.
New apsidal motion determination of eccentric eclipsing binary V 1143 Cyg
Inf.Bull.Var.Stars N4363,1-4.

[43] Hill G. and Fisher W.A.
Studies of early-type varieble stars.II. The orbit and masses ofHR 7551
Astron.Astrophys. **139** (1985) 123-131.

[44] Martynov D.Ya. and Khaliulilin Kh.F.
On the relativistic motion of periastron in the eclipsing binary system DI Her
Astrophys.and Space Sci. **71** (1980) 147-170.

[45] Popper D.M.
Rediscussion of eclipsing binaries.XVII. DI Herculis, a B-tipe system with an accentric orbit
Astron.J. **254** (1982) 203-213.

[46] Martynov D.Ia. Lavrov M.I.
Revizia elementov fotometricheskoi orbity i skorosti vrashenia linii apsid u zatmennoi dvoinoi sistemy DI Her
Pisma v Astron.Zh. **13** (1987) 218-222 (in Russian).

[47] Khaliulilin Kh.F., Khodykin S.A. and Zakharov A.I.
On the nature of the anomalously slow apsidal motion of DI Herculis
Astrophys.J. **375** (1991) 314-320.

[48] Khaliulilina A.I. and Khaliulilin Kh.F.
Vrashenie linii apsid v zatmennoi dvoinoi sisteme HS Her
Astron.cirk.N 1552 (1992) 15-16(in Russian).

[49] Martynov D.Ia., Voloshina I.E. and Khaliulilina A.I.
Fotometricheskie elementy zatmennoi sistemy HS Her
Asrton. zh. **65** (1988) 1225-1229 (in Russian).

[50] Mezzetti M., Predolin F., Giuricin G. and Mardirossian F.
Revised photometric elements of eight eclipsing binaries
Astron.Astroph.Suppl. **42** (1980) 15-22.

[51] Mossakovskaya L.V. and Khaliulilin Kh.F.
Tret'e telo v zatmennoi sisteme s apsidalnym dvizheniem CO Lac?
Astron. cirk.N1495, 5-6 (in Russian).

[52] Semeniuk I.
Apsidal motion in binary systems. I. CO Lacertae, an eclipsing variable with apsidal motion
Acta Astron. **17** (1967) 223-224.

[53] Andersen J.
Accurate masses and radii of normal stars
Astron.Astroph.Rev. **3** (1991) 91-126.

[54] Khaliulilina A.I., Khaliulilin Kh.F. and Martynov D.Ya.
Apsidal motion and the third body in the system RU Monocerotis
Montly .Not.Roy.Astron.Soc. **216** (1985) 909-922.

[55] Martynov D.Ya. and Khaliulilina A.I.
RU Monocerotis: poslednie resultaty
Astron.zh.**63** (1986) 288-297 (in Russian).

[56] Shneller H.
Uber die periodenanderrungen bei bedeckungsveranderlichen
Budd.Mitt. N53 (1962) 3-41.

[57] Kort S.J., J. de, *The orbit and motion of priastron of GN Normae*
Ricerche Astron. **3** (1954) 119-128.

[58] Kamper B.C.
Light-time orbit and apsidal motion of eclipsing binary U Ophiuchi
Astrophys. Space Sci. **120** (1986) 167-189.

[59] Clausen J.V., Gemenez A. and Scarfe C.
Absolute dimentions of eclipsing binaries.XI. V 451 Ophiuchi
Astron.Astroph. **167** (1986) 287-296.

[60] Khaliulilin Kh.F. and Kozyreva V.S.
Photometric light curves and physical parameters of eclipsing binary systems IT Cas, CO Cep, AI Hya with possible apsidal motions
Astrophys. and Space Sci., **155** (1989) 53-69.

[61] Monet D.G.
A discussion of apsidal motion detected in selected spectroscopic binary systems
Astrophys. J., **237** (1980) 513-528.

[62] Svechnicov M.A.
Katalog orbitalnyh elementov, mass i svetimostei tesnyh dvoinyh zvezd
Irkutsk, Izd-vo Irkutsk. Univer.(In Russian).

[63] Brancewicz H.K. and Dworak T.Z.
A Catalogue of parameters for eclipsing binaries
Acta Astron., **30** (1980) 501-524.

[64] Wolf M. and Saronova L.,
Aspidal motion in the eclipsing binary FT Ori
Astron.Astroph. Suppl.**114** (1995) 143-146.

[65] Drozdz M., Krzesinski J. and Paydosz G.,
Aspidal motion of IQ Persei
Inf. Bull.Var.Stars, N3494, 1-4.

[66] Lacy C.H.S. and Fruch M.L.
Absolute dimentions and masses of eclipsing binaries. V. IQ Persei
Astroph.J.**295** (1985) 569-579.

[67] Andersen J.
Spectroscopic observations of eclipsing binaries.V. Accurate mass determination for the B-type systems V 539 Arae and ξ Phaenicis
Astron.Astroph.**118** (1983) 255-261.

[68] Odell A.P.
The structure of Alpha Virginis.II. The apsidal constant
Astroph.J.**192** (1974) 417-424.

[69] Gronbech B.
Four-color photometry of eclipsing binaries.V. photometric elements of NO Puppis
Astron.Astroph.**50** (1980) 79-84.

[70] Harmanec P.
Stellar masses and radii based on motion binary data
Bull.Astron.Inst.Czech.**39** (1988) 329-345.

[71] Andersen J., Clausen L.V. and Nordstrom B.
Absolute dinemtions of eclipsing binaries.V. VV Pyxidis a detached early A-tipe system with equal components
Astron.Astroph.**134** (1984) 147-157.

[72] Lacy C.H.S.
The photometric orbit and apsidal motion of YY Sagittarii
Astroph.J.**105** (1993) 637-645.

[73] Lacy C.H.S.
The photometric orbit and apsidal motion of V 523 Sagittarii
Astroph.J.**105** (1993) 630-636.

[74] Lacy C.H.S.
The photometric orbit and apsidal motion of V 526 Sagittarii
Astroph.J.**105** (1993) 1096-1102.

[75] Andersen J. and Gimenes A.
Absolute dinemtions of eclipsing binaries.VII. V 1647 Sagittarii
Astron.Astroph.**145** (1985) 206-214.

[76] Swope H.H.
V 2283 Sgr, an eclipsing star with rotating apse
Ric.Astron.**8** (1974) 481-490.

[77] O'Konnell D.J.K.
The photometric orbit and apsidal motion of V2283 Sagittarii
Ric.Astron.**8** (1974) 491-497.

[78] Andersen J., Clausen L.V., Nordstrom B. and Popper D.M.
Absolute dinemtions of eclipsing binaries.VIII. V 760 Scorpii
Astron.Astroph.**151** (1985) 329-339.

[79] Clausen L.V., Gimenez A. and Houten C.J.
Four-color photometry of eclipsing binaries.XXVII. A photometric anallysis of the (possible) Ap system AO Velorum
Astron.Astroph.**302** (1995) 79-84.

[80] Popper D.M.
Stellar masses
Ann. Rev. Astron. and Astroph.**18** (1980) 115-164.

[81] Dukesr R.J.
The beta Cephei nature of Spica
Astroph.J.**192** (1974) 81-91.

[82] Khaliulilina A.I.
DR Vulpeculae: the quadruple system
Montly .Not.Roy.Astron.Soc. **225** (1987) 425-436.

[83] Khaliulilin Kh.F. and Khaliulilina A.I.
Fotometricheskoe issledovanie zatmennoi zvezdy DR Vul. Parametry sistemy i vrashenie linii apsid,
Astron.zh., **N65** (1988) 108-116 (in Russian).

[84] Khaliulilin Kh.F. and Kozyreva V.S.
Photometric light curves and physical parameters of eclipsing binary systems IT Cas, CO Cep, AI Hya with possible apsidal motions
Astrophys. and Space Sci., **155** (1989) 53-69.

[85] Holmgren D. anf Wolf M. *Apsidal motion of the eclipsing binary IT Cassiopeiae*
Observatory **116** (1996) 307-312.